Original paper

Parallel Algorithms for Computational Models of Geophysical Systems

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Resumen

Los modelos matemáticos de muchos sistemas geofísicos requieren el procesamiento de sistemas algebraicos de gran escala. Las herramientas computacionales más avanzadas están masivamente paralelizadas. El software más efectivo para resolver ecuaciones diferenciales parciales en paralelo intenta alcanzar el *paradigma de los métodos de descomposición de dominio*, que hasta ahora se había mantenido como un anhelo no alcanzado. Sin embargo, un grupo de cuatro algoritmos –los *algoritmos DVS*- que lo alcanzan y que tiene aplicabilidad muy general se ha desarrollado recientemente. Este artículo está dedicado a presentarlos y a ilustrar su aplicación a problemas que se presentan frecuentemente en la investigación y el estudio de la Geofísica.

Palabras clave: computational-geophysics, computational-PDEs, non-overlapping DDM, BDDC; FETI-DP.

Abstract

Mathematical models of many geophysical systems are based on the computational processing of large-scale algebraic systems. The most advanced computational tools are based on massively parallel processors. The most effective software for solving partial differential equations in parallel intends to achieve the *DDM-paradigm*. A set of four algorithms, the *DVS-algorithms*, which achieve it, and of very general applicability, has recently been developed and here they are explained. Also, their application to problems that frequently occur in Geophysics is illustrated.

Key words: computational-geophysics, computational-PDEs, non-overlapping DDM, BDDC, FETI-DP.

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1. Introduction

Mathematical models of many systems of interest, including very important continuous systems of Earth Sciences and Engineering, lead to a great variety of partial differential equations (PDEs) whose solution methods are based on the computational processing of large-scale algebraic systems. Furthermore, the incredible expansion experienced by the existing computational hardware and software has made amenable to effective treatment problems of an ever increasing diversity and complexity, posed by scientific and engineering applications [PITAC, 2006].

Parallel computing is outstanding among the new computational tools and, in order to effectively use the most advanced computers available today, massively parallel software is required. Domain decomposition methods (DDMs) have been developed precisely for effectively treating PDEs in parallel [DDM Organization, 2012]. Ideally, the main objective of domain decomposition research is to produce algorithms capable of *'obtaining the global solution by exclusively solving local problems'*, but up-to-now this has only been an aspiration; that is, a strong desire for achieving such a property and so we call it *'the DDM-paradigm'*. In recent times, numerically competitive DDM-algorithms are *non-overlapping*, *preconditioned* and necessarily incorporate *constraints* [Dohrmann, 2003; Farhat *et al*., 1991; Farhat *et al.*, 2000; Farhat *et al.*, 2001; Mandel, 1993; Mandel *et al.*, 1996; Mandel and Tezaur, 1996; Mandel *et al.*, 2001; Mandel *et al.*, 2003; Mandel *et al*., 2005; J. Li *et al*., 2005; Toselli *et al*., 2005], which pose an additional challenge for achieving the *DDM-paradigm*.

Recently a group of four algorithms, referred to as the *'DVS-algorithms'*, which fulfill the *DDM-paradigm*, was developed [Herrera *et al*., 2012; L.M. de la Cruz *et al*., 2012; Herrera and L.M. de la Cruz *et al*., 2012; Herrera and Carrillo-Ledesma *et al*., 2012]. To derive them a new discretization method, which uses a non-overlapping system of nodes (the *derived-nodes*), was introduced. This discretization procedure can be applied to any boundary-value problem, or system of such equations. In turn, the resulting system of discrete equations can be treated using any available DDM-algorithm. In particular, two of the four *DVS-algorithms* mentioned above were obtained by application of the well-known and very effective algorithms BDDC and FETI-DP [Dohrmann, 2003; Farhat *et al*., 1991; Farhat *et al*., 2000; Farhat *et al.*, 2001; Mandel *et al.*, 1993; Mandel *et al.*, 1996; Mandel and Tezaur, 1996; Mandel *et al.*, 2001; Mandel *et al.*, 2003; Mandel *et al*., 2005; J. Li *et al.*, 2005; Toselli *et al.*, 2005]; these will be referred to as the *DVS-BDDC* and *DVS-FETI-DP* algorithms. The other two, which will be referred to as the *DVS-PRIMAL* and *DVS-DUAL* algorithms, were obtained by application of two new algorithms that had not been previously reported in the literature [Herrera *et al*., 2011; Herrera *et al*., 2010; Herrera *et al*., 2009; Herrera *et al.*, 2009; Herrera, 2008; Herrera, 2007]. As said before, the four *DVS-algorithms* constitute a group of preconditioned and constrained algorithms that, for the first time, fulfill the *DDM-paradigm* [Herrera *et al*., 2013; L.M. de la Cruz *et al.*, 2012].

Both, BDDC and FETI-DP, are very well-known [Dohrmann, 2003; Farhat *et al*., 1991; Farhat *et al*., 2000; Farhat *et al*., 2001; Mandel *et al.*, 1993; Mandel *et al.*, 1996; Mandel and Tezaur, 1996; Mandel *et al*., 2001]; and both are highly efficient. Recently, it was established that these two methods are closely related and its numerical performance is quite similar [Mandel *et al*., 2003; Mandel *et al*., 2005]. On the other hand, through numerical experiments, we have established that the numerical performances of each one of the members of *DVS-algorithms* group (*DVS-BDDC*, *DVS-FETI-DP*, *DVS-PRIMAL* and *DVS-DUAL*) are very similar too. Furthermore, we have carried out comparisons of the performances of the standard versions of BDDC and FETI-DP with *DVS-BDDC* and *DVS-FETI-DP*, and in all such numerical experiments the DVS algorithms have performed significantly better.

Each *DVS-algorithm* possesses the following conspicuous features:

- It fulfills the *DDM-paradigm*;
- It is applicable to symmetric, non-symmetric and indefinite matrices (i.e., neither positive, nor negative definite); and
- It is preconditioned and constrained, and has update numerical efficiency.

Furthermore, the uniformity of the algebraic structure of the matrix-formulas that define each one of them is remarkable.

This article is organized as follows. In Section 2 the basic definitions for the DVS framework are given; here we define the set of 'derived-nodes', internal, interface, primal and dual nodes, the 'derived-vector-space', among others. Section 3 is devoted to define the new set of vector spaces that conforms the DVS framework; the Euclidean inner product, is also defined here. In Section 4 the 'transformed-problem' on the derived-nodes is explained in detail, and this is our starting point to define the DVS algorithms. Section 5 presents a summary of the four DVS-algorithms: DVS-BDDC, DVS-FETI-DP, DVS-PRIMAL and DVS-DUAL. In Section 6 we give the numerical procedures