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Modeling and simulation of GPR wave propagation through wet snowpacks: Testing the sensitivity of a method for snow water equivalent estimation

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ABSTRACT

Snow water equivalent (SWE) of a snowpack is an important input to distributed snow hydrological models used for runoff predictions in areas with annual snowpacks. Since the conventional method of manually measuring SWE is very time-consuming, more automated methods are being adopted, such as using ground penetrating radar operated from a snowmobile with SWE estimated from radar wave two-way travel time. However, this method suffers from significant errors when liquid water is present in the snow. In our previous work, a new method for estimating SWE of wet snowpacks from radar wave travel times and amplitudes was proposed, with both these parameters obtained from a common mid-point survey. Here we present a custom ray-based model of radar wave propagation through wet snowpacks and results of MATLAB simulations conducted to investigate the method's sensitivity to measurement errors and snowpack properties. In particular, for a single-layer snowpack up to 2.1 m deep and with liquid water content up to 4.5% (by volume), the simulations indicate that SWE can be estimated with an error of ±5% or less if (a) the noise (measurement errors) in resulting amplitude has a standard deviation less than 15% and(b) the noise in two-way travel time has a standard deviation less than 0.075 ns (22.5% and 0.15 ns for a snowpack less than 1.3 m deep).

1. Introduction

Reliable estimates of snow water equivalent (SWE) over large areas constitute an important input to distributed snow hydrological models used for predicting snowmelt runoffs (see examples of models in Arheimer et al. (2008); Kolberg and Gottschalk (2006); Udnæs et al. (2007); Taurisano et al. (2007)). Such predictions are essential for hydropower industry, as an efficient energy production in areas where snow is a substantial part of total precipitation is dependent on accurate runoff forecasts. SWE data over large areas can also be used for other applications in hydrological and glaciological research (e.g., mass balance studies of glaciers and polar ice caps, Spikes et al. (2004)).

Estimates of SWE over large areas can be obtained from remote sensing data gathered from satellites or aircrafts. These methods typically suffer from low accuracy and resolution problems, hence they need to be validated and calibrated with "ground-truth" measurements, which can be conducted to obtain the distribution of SWE over time or in space (Lundberg et al., 2010).

The traditional method for obtaining spatial distribution of snow data is to conduct manual point measurement of SWE following lines traversing a representative terrain of the area of interest (so-called snow courses) (Singh and Singh, 2001). This method, however, is very time-consuming and so more automated measurement methods have gained popularity in recent years.

One semi-automatic method involves using ground penetrating radar (GPR) operated from a snowmobile. Here SWE is estimated via empirical formulas from the two-way travel time, i.e., the time it takes a radar wave to travel from the transmitter through the snowpack and back to the receiver (see, e.g., Andersen et al. (1987); Sand and Bruland (1998)). However, if liquid water is present in the snow, this method suffers from substantial errors when liquid water content is unknown (Lundberg and Thunehed, 2000).

To address this problem, at least two different methods for estimating liquid water content have been proposed. Both methods rely on using a multi-channel radar system to conduct a common mid-point survey of the snowpack at each point along a measurement profile. The first method was proposed by Bradford (Bradford and Harper, 2006; Bradford et al., 2009) and relies on measuring frequency-dependent attenuation of radar waves.

In the other method, proposed by the first author of this paper (Granlund, 2009), the resulting radar wave amplitude is measured and liquid water content and, thereby, SWE are determined from radar wave attenuation and two-way travel time. However, this method involves quite a large number of calculation steps, which

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may make it sensitive to measurement errors, and the magnitude of this sensitivity is difficult to estimate.

Here we investigate this method's sensitivity to different measurement errors, snowpack properties, and characteristics of the terrain. To this end, we suggest to model radar wave propagation through snowpacks of varying depth and structure. To be able to calculate reflectivity of the snow/ground interface, both the shape of the ground surface and the electrical permittivity of the top ground layer are defined as parameters of the model. Measurement errors are modeled by adding noise to the calculated values of two-way travel time and resulting amplitude. These “noisy” values are then used as inputs to the SWE estimation method, and the obtained estimates of SWE are compared to the values calculated in the model, allowing us to draw conclusions on the method's applicability and on the effect that measurement errors have on its accuracy.

A number of numerical methods for modeling GPR data have been proposed, such as ray-based methods (Cai and McMechan, 1995), frequency-domain methods (Powers and Olhoeft, 1994), and finite-difference time-domain techniques (Irving and Knight, 2006). Our model is based on Cai and McMechan's ray-based model, but it has been modified and extended to accommodate the specific requirements discussed above. This model was chosen because it is relatively simple and easy to adapt. At the same time, it should be accurate enough to provide preliminary results about the applicability of the proposed method, with final verification conducted using field tests (which will be presented in a separate paper).

In this paper, we present the method for SWE estimation first proposed in Granlund (2009), the model of radar wave propagation through a wet snowpack, and the results of MATLAB simulations of the model.

2. Suggested method for estimating SWE of wet snowpacks

The method for estimating SWE of wet snowpacks, suggested in the licentiate thesis by Granlund (2009), relies on a common mid-point survey conducted for each measurement point, with radar antennas positioned directly on the snow surface. From radar data collected in the survey, both resulting amplitude and two-way travel time for at least two radar wave paths can be obtained.[2] We further assume that the effective source amplitude A_0 (-) has been established with a reference measurement and that the directivity of the transmitter(s) D_T (-) and the receiver(s) D_R (-) are known as functions of the angle of transmission and reception φ (*rad*). With antennas positioned directly on the snow surface, directivity is also dependent on relative effective electrical permittivity of snow ε_{snow} (here and below ε denotes *relative* electrical permittivity).

The steps of the procedure for estimating SWE are described below and presented schematically in Fig. 1.

Step 1. A common mid-point survey allows us to determine snowpack depth h_{snow} (m) and relative effective electrical permittivity of snow ε_{snow} from radar wave two-way travel times *twt* (s) and distances between the antennas S (m). In the common mid-point method, the snowpack is treated as single-layer and the snow and ground surfaces are assumed to be parallel. Under these assumptions, we also obtain the travel path length d (m) and the angle ϕ (*rad*) for each travel path of radar pulses; a single angle characterizes the angles of transmission and reception as well as the incidence angle at the snow/ground interface.

Step 2. For radar antennas placed on the snow surface, the measured amplitude of a reflected wave can be expressed as (Cai and McMechan, 1995):

$$A = \frac{A_0 D_T D_R R}{G} e^{-\alpha d}, \text{ where } \alpha = \frac{\sigma_{snow}}{2}\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{snow}}} \quad (1)$$

Here G is geometrical spreading (-), R is reflectivity of the snow/ground interface (-), σ_{snow} is effective electrical conductivity of snow (S/m), μ_0 is magnetic permeability (H/m) and ε_0 is electrical permittivity (F/m) of free space. The effective values of conductivity and permittivity have to be used since the snowpack may not be homogeneous.

In a homogeneous snowpack, the amplitude of a spherical wave decreases as r^{-1} where r is the radius of the sphere (Cai and McMechan, 1995), so the geometrical spreading term G is equal to the travel path length d. Further, reflectivity can be expressed from Fresnel equations using Snell's law. The transmitter and receiver antennas should be placed so that radar waves are either s-polarized (with electrical field vector perpendicular to the plane of incidence) or p-polarized (with electrical field vector parallel to the plane of incidence), and then for an incidence angle ϕ (*rad*) we have:

$$R = \frac{\cos\phi - \sqrt{\frac{\varepsilon_{ground}}{\varepsilon_{snow}} - \sin^2\phi}}{\cos\phi + \sqrt{\frac{\varepsilon_{ground}}{\varepsilon_{snow}} - \sin^2\phi}} \quad (2)$$

for an s-polarized wave and

$$R = \frac{\cos\phi - \sqrt{\frac{\varepsilon_{ground}}{\varepsilon_{snow}} - \frac{\varepsilon^2_{ground}}{\varepsilon^2_{snow}} \cdot \sin^2\phi}}{\cos\phi + \sqrt{\frac{\varepsilon_{ground}}{\varepsilon_{snow}} - \frac{\varepsilon^2_{ground}}{\varepsilon^2_{snow}} \cdot \sin^2\phi}} \quad (3)$$

for a p-polarized wave. Here ε_{ground} is relative electrical permittivity of the top ground layer.

Reflectivity can be substituted into Eq. (1), which yields one equation with two unknowns: effective electrical conductivity of snow and electrical permittivity of the ground. Considering two radar pulses reflected from the same point on the ground with different angles of incidence, we obtain a system of two equations with two unknowns, which can be solved numerically.

Step 3. Volumetric liquid water content of the snowpack θ_{water} (*vol.%*) can now be determined from the experimentally established relationship between effective electrical conductivity of snow σ_{snow} (S/m) and snow wetness (Granlund et al., 2010):

$$\sigma_{snow} = 0.001 + 0.3 \cdot \theta_{water} \quad (4)$$

Here and everywhere below, θ_{water}, θ_{ice}, and θ_{air} are volumetric content of water, ice, and air, respectively. In formulas, they are used as factors, i.e., values between 0 and 1.

Step 4. Relative effective electrical permittivity of snow together with snow wetness can be substituted into Looyenga's empirical formula for mixtures (Looyenga, 1965):

$$\sqrt[3]{\varepsilon_{snow}} = \theta_{ice} \cdot \sqrt[3]{\varepsilon_{ice}} + \theta_{water} \cdot \sqrt[3]{\varepsilon_{water}} + \theta_{air} \cdot \sqrt[3]{\varepsilon_{air}} \quad (5)$$

Relative electrical permittivity of ice ε_{ice}, water ε_{water}, and air ε_{air} are known physical constants for a specific radar frequency and temperature (the temperature of wet snow can be assumed to be 0 °C). Since $\theta_{air} = 1 - \theta_{water} - \theta_{ice}$, we can

[2] Using more than two measurements for each point results in an over-determined system of equations.