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River and lake ice thickening, thinning, and snow ice formation

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ABSTRACT

There is probably no more important characteristic of river and lake ice than the thickness. This paper consists of three parts: In Part I the thickening of ice via conduction is analyzed and tested against an extensive data set assembled by Bilello (1961–1996) that included weekly measurements of ice thickness and snow on the ice for a number of sites in Alaska and Canada. It is shown that the largest variations from year to year at a given site are associated with the thickness of the snow on the ice, and secondarily by variations in the coldness of the winter period of thickening. If no snow ice forms, quite good agreement with a simple thickening algorithm is achieved. This leads to Part II where the formation of snow ice is analyzed but the analysis is constrained by the fact that sometimes, even though the weight of the snow is enough to submerge the solid ice cover, there seems to be no snow ice formation. Differences between the formation of snow ice on river and lake ice covers are examined. For river ice covers, the water from below cannot always find a path to the top of the ice cover. Finally, in Part III the thinning of river ice covers is analyzed using a simple algorithm based on solar radiation calculations and sensible and latent heat transfers from the air to the ice. Good agreement is obtained with the extensive data set of Bilello (1980).
Implications for practical calculations of thickness are discussed.

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1. Introduction

This paper was motivated by a number of new findings associated with the thickening, thinning, and snow ice formation on rivers and lakes. These include the finding that snow on the ice may be reasonably approximated by examining the snow on the ground adjacent to a river or lake; the finding that even when conditions are conducive to snow ice formation, it does not always occur particularly on rivers; a relationship for snow cover density on the ice cover; the finding that when the snow cover depletes due to flooding, it often "overdepletes"; and a fairly comprehensive but simple analysis of the thinning of ice to include a modification of the sensible components together with addition of the radiation components. It is not meant to be a comprehensive review paper of the many contributions to our understanding of river and lake ice formation. The emphasis is on analysis of the thickening and thinning using the most usually available data which is most often only the air temperature and precipitation records at or near the site of interest.

2. Part I—thickening of river and lake ice

2.1. Previous work

Most analyses of the thickening of ice covers are performed using variations of the Stefan formulation in which the thickness is proportional to the square root of the accumulation of degree-days of freezing and is believed to have been originated by Stefan (1889). Michel (1971) elaborated on the Stefan analysis to include the effect of the insulating effect of the snow thickness. Ashton (1980, 1986) further expanded the analysis to include the thermal resistance of the surface to air boundary layer resistance. Shen and Yapa (1985) further examined the influences of the snow cover thickness. There have been many other studies that implement the Stefan result and determine the empirical coefficient that enables agreement with field data. Michel (1971) summarizes much of that experience. It is recognized that detailed energy budget and thickening analyses may be made on an hourly basis with detailed input data on air temperature variation, wind speed, and the other pertinent variables. Launiainen and Cheng (1998) provide an excellent example of such analyses. Also of interest are the papers of Duguay et al. (2003) and of Flato and Brown (1996). The intent here is to provide the basis for more simplified calculations of thickening and thinning using daily values of average air temperature.

2.2. Analysis

The dominant thermal gradients that result in heat conduction in ice covers are vertical. The one-dimensional heat conduction in the ice cover is given by

$$\rho_i c_{pi} \frac{\partial}{\partial t} T_i = \frac{\partial}{\partial z}\left(k_i \frac{\partial}{\partial z} T_i + \varphi\right) \tag{1}$$

where T_i is the temperature in the ice (always ≤0 °C), ρ_i is the density of the ice, c_{pi} is the specific heat capacity of the ice, k_i is the thermal conductivity of the ice, and φ is an internal heat source term due to solar radiation absorption. This equation may be solved by finite-difference techniques. For quasi-steady conditions ($\frac{\partial}{\partial t}T_i = 0$), and neglecting φ (which is important only during the thinning period) the equation reduces to the familiar steady state heat conduction equation

$$\Phi_i = k_i \, dT_i / dz \tag{2}$$

where Φ_i is the flux of heat through the ice. The boundary conditions are that at the bottom the temperature is always at the melting point T_m and at the top surface of the ice the temperature is T_s so that $\Phi_i = -k_i\ (T_m - T_s)/h$ where h_i is the thickness of the ice. At the bottom surface the flux of heat into the ice equals the rate of ice production

$$dh_i / dt = [1/(\rho_i \lambda)](\Phi_i - \Phi_{wi}) = [k_i/(\rho_i \lambda)](T_m - T_s)/h_i] - \Phi_{wi}/(\rho_i \lambda) \tag{3}$$

where Φ_{wi} is the flux of heat from the water to the undersurface. Neglecting Φ_{wi} and integrating with respect to time then yields the growth equation in the form

$$h_i = [2\ k_i/(\rho_i \lambda)]^{1/2}[(T_m - T_s)t]^{1/2}. \tag{4}$$

If T_s is assumed equal to the air temperature T_a (not true generally due to boundary layer effects), the well-known Stefan equation for ice thickening results in the form

$$h_i = \alpha[2\ k_i/(\rho_i \lambda)]^{1/2} S^{1/2} \tag{5}$$

where S is the summation of accumulated days of freezing temperatures (1 freezing degree day is −1 °C over one day; 1 freezing degree day = 86,400 °C s), and α is an empirical coefficient (always less than 1.0) to account for a myriad of incorrect assumptions, the most important of which are that $T_s = T_a$, neglect of the insulating effect of a