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Cold Regions Science and Technology 54 (2008) 19–35
cold regions science and technology
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Theoretical modeling framework for an unsaturated freezing soil

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Received 9 July 2006; accepted 3 December 2007

Abstract

Based on the authors' theoretical modeling framework of the saturated freezing soil, this paper further discusses the air phase in the unsaturated freezing soil in two ideal situations: air in the soil is linked to the atmosphere (the open porous medium) and air in the soil is isolated from the atmosphere (the sealed porous medium). The corresponding theoretical modeling framework for a multi-phase porous medium with interaction of water, heat and deformation is established in this paper. The proposed theoretical model is then extended to the general case (unsaturated half-open and half-sealed porous medium). Also, a finite element numerical solution to the heat-moisture-deformation coupling behavior of the unsaturated freezing soil is obtained. The computer software for solving this problem is also developed. The in-situ measurements for Hua Shixia highway roadbed located in Qinghai–Tibetan Plateau are introduced to compare with the numerical results obtained by the proposed model. The measurement results indicate that the numerical results for the temperature field variations with time are found to be in good correlation with the roadbed depth, just as the relative freezing deformations are correlated with road surface. Numerical modeling results, when compared to field measurements of temperature and deformation, are within 10 to 20%. The comparative analysis shows that proposed theoretical modeling framework and its numerical solution for the temperature–moisture-deformation coupling behavior is suitable and acceptable.

Keywords: Freezing soil; Unsaturated; Frost heave; Thaw settlement; Modeling framework; FEM model; Quasi-confined media

1. Introduction

In the U.S. Alaska, Russian Siberia and on the Qinghai Tibetan Plateau of China, there have been many engineering problems associated with frost heave and thawing settlement in the construction of the highway, railway and building foundations. At the early stage, the empirical formulas were adopted to predict frost heave and thawing settlements. As a result, various freezing models for coupled heat and moisture flow and their numerical solutions were developed in the 1970s. The physical and theoretical basis for heat and mass transfer in frost models has been reviewed in many papers and technical reports (Kay and Perfect, 1988; Shen and Ladanyi, 1987; Sheng, 1994; Kujala, 1997). All these models can be classified into four different groups, i.e.,

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doi:10.1016/j.coldregions.2007.12.001

hydrodynamic models (Harlan, 1973; Sheppard et al., 1978; Jansson and Halldin, 1979; Fukuda, 1982; Fukuda and Nakagawa, 1985; Guymon et al., 1980, 1993), rigid-ice models (Konrad and Morgenstern, 1980; Gilpin, 1980; O'Neill and Miller, 1985; Nixon, 1991), thermo-mechanical models (Duquennoi and Fremond, 1989; Fremond and Mikkola, 1991; Miao et al., 1999) and semi-empirical models, e.g., segregation potential models (Kujala, 1997).

Based on force equilibrium and the principles of mass and energy conservation for a three phase (soil particle, ice and water) saturated porous medium, Li et al. (2000a,b) and Li et al. (2002) developed a theoretical modeling framework for heat-moisture-deformation coupling behaviors of freezing soil and jointed rock. In the proposed theoretical model, the different force interaction behaviors between the soil skeleton and ice particles in the frozen zone are taken into consideration. However, most frozen soils are unsaturated; and the soil thermal/mechanical properties as a whole vary greatly with the effective stresses which may change significantly with the pore air pressure inside the unsaturated soil. Particularly, the freezing or thawing situation might result in great changes with pore water pressure.

The thickness of ice lenses varies from several millimeters to meters with the temperature change from −1 °C to −30 °C. When the ice lenses are very thick, the air in the thick ice can be considered to isolate from the atmosphere. In thawing soils, the air in the soil may be open to the atmosphere for shallow sands, while in deep clay, the air can be assumed as closed to atmosphere approximately. As far as the representative unit discussed in this paper is concerned, when the ice lenses are thin and homogeneously distributed in the representative unit, some air may be isolated from the atmosphere while other air may be open to the atmosphere, so this kind of soil can be assumed as a type of half-open and half-sealed porous medium.

This paper continues our earlier research work and focuses on a four-phase unsaturated porous media consisting of soil particle, ice, water and air under freezing and thawing conditions. Two ideal situations of the porous features of freezing soil are considered: one is that the air in the soil is open to the atmosphere (e.g. air in the shallow sand), while the other is that air in the soil is isolated from the atmosphere (e.g. for the most deep clay). In reality, air that exists in the unsaturated frozen soil is between the two ideal conditions described above; and it is neither completely open to the atmosphere nor completely isolated from the atmosphere. Accordingly, it is very difficult to determine the real field conditions for the air in the soils, so we can assume that the soil condition is a mixture body between the two cases and therefore a half-opened and half-sealed frozen soil is examined here. At the same time, we assume that air flow satisfies Darcy's law.

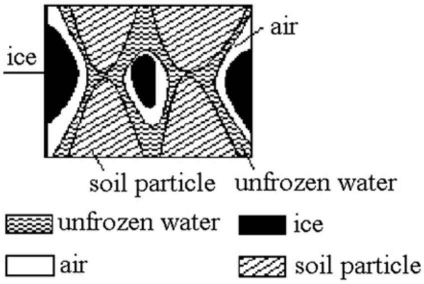

Fig. 1. A representative unit cross-section of an unsaturated, frozen soil.

2. Effective stress principle for the unsaturated freezing soil

The stresses acting on all components of the unsaturated freezing soil unit are: soil particles with contacting stress σ^s, s is the superscript convenient for expressing tensor format, ice particles stress σ^i, pore water pressure of unfrozen water p_w, and air pressure in soil voids p_a. Considering a freezing soil unit with a cross-sectional area A, the actual soil particles contacting area is A_s, the unfrozen water contacting area is A_w, the ice particles contacting area is A_i and the air contacting area is A_a, as shown in Fig. 1. Accordingly, we obtain the following formula:

$$A = A_s + A_w + A_i + A_a \tag{1}$$

The total stress can be expressed as:

$$\sigma_{ij} = \frac{A_s}{A}\sigma^s_{ij} + \frac{A_i}{A}\sigma^i_{ij} + \frac{A_w}{A}p_w\delta_{ij} + \frac{A_a}{A}p_a\delta_{ij} \tag{2}$$

where δ_{ij} is Kronecker's delta = 0 for $i \neq j$ and = 1 for $i = j$. In Eq. (2), the relationship between the average effective stress on the soil skeleton σ'_{ij} and the actual contacting stress on the soil particles σ^s_{ij} can be written as:

$$\sigma'_{ij} = \frac{A_s}{A}\sigma^s_{ij} \tag{3}$$

Usually, the actual ratio between the soil particle contacting area and the cross-sectional area is very small in comparison with other three items so that it can be neglected for simplicity. Therefore, we obtain:

$$\frac{A_w}{A} + \frac{A_i}{A} + \frac{A_a}{A} = 1 \tag{4}$$