Earth and Planetary Science Letters 436 (2016) 121–129

Contents lists available at ScienceDirect

Earth and Planetary Science Letters

www.elsevier.com/locate/epsl

Tests of diffusion-free scaling behaviors in numerical dynamo datasets

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ARTICLE INFO

Article history:
Received 22 July 2015
Received in revised form 22 November 2015
Accepted 14 December 2015
Available online 30 December 2015
Editor: B. Buffett

Keywords:
fluid dynamics
dynamo simulations
heat transfer
non-dimensional scalings
outer core
rotating convection

ABSTRACT

Many dynamo studies extrapolate numerical model results to planetary conditions by empirically constructing scaling laws. The seminal work of Christensen and Aubert (2006) proposed a set of scaling laws that have been used throughout the geoscience community. These scalings make use of specially-constructed parameters that are independent of fluid diffusivities, anticipating that large-scale turbulent processes will dominate the physics in planetary dynamo settings. With these 'diffusion-free' parameterizations, the results of current numerical dynamo models extrapolate directly to fully-turbulent planetary core systems; the effects of realistic fluid properties merit no further investigation. In this study, we test the validity of diffusion-free heat transfer scaling arguments and their applicability to planetary conditions. We do so by constructing synthetic heat transfer datasets and examining their scaling properties alongside those proposed by Christensen and Aubert (2006). We find that the diffusion-free parameters compress and stretch the heat transfer data, eliminating information and creating an artificial alignment of the data. Most significantly, diffusion-free heat transfer scalings are found to be unrelated to bulk turbulence and are instead controlled by the onset of non-magnetic rotating convection, itself determined by the viscous diffusivity of the working fluid. Ultimately, our results, in conjunction with those of Stelzer and Jackson (2013) and King and Buffett (2013), show that diffusion-free scalings are not validated by current-day numerical dynamo datasets and cannot yet be extrapolated to planetary conditions.

1. Introduction

The Earth's magnetic field is generated by dynamo action in the liquid metal outer core. Much of our understanding of the geodynamo comes from direct numerical simulations that solve the governing equations of magnetohydrodynamic flow in a rotating spherical shell of conducting fluid (e.g., Glatzmaier, 2013). Numerical models attempt to reproduce key features of the observed field, such as the dipolar morphology and polarity reversals, and link these features to flow behaviors (e.g., Glatzmaier and Coe, 2007; Aubert et al., 2008; Olson et al., 2011). However, limitations in computational power preclude these models from being run with parameter values similar to those that exist in the core. Applying the results of models to the core therefore requires massive extrapolation through parameter space. A prevalent method to achieve this extrapolation is to empirically determine power law scaling relations between the governing parameters. These scaling laws are constructed by conducting large surveys of dynamo models within achievable parameter ranges. These scalings are then extrapolated from presently-accessible parameter values to planetary-scale estimates.

In the fluid physics community, the so-called ultimate regime of convective turbulence has been theorized and searched for extensively (e.g., Kraichnan, 1962; Spiegel, 1971; Ahlers et al., 2009; Roche et al., 2010; Grossmann and Lohse, 2011; Julien et al., 2012b; He et al., 2014). In this ultimate regime, it is hypothesized that the fluid's molecular diffusivities do not contribute meaningfully to the physics, and instead only macro-scale turbulent phenomena dictate the flow behaviors. In order to extrapolate dynamo results to the core, Christensen and Aubert (2006) aimed to produce analogous asymptotic scalings for dynamo physics. These scaling laws have been widely used in the recent geoscience literature (e.g., Aubert et al., 2009, 2010; Tarduno et al., 2012; Driscoll and Bercovici, 2013), the planetary science literature (e.g., Christensen, 2002; Hauck et al., 2006; Olson and Christensen, 2006; Aurnou, 2007; Takahashi et al., 2008; Christensen et al., 2009, 2010; Schmitz and Tilgner, 2010; Weiss et al., 2010; Aurnou and Aubert, 2011; Christensen, 2011; Showman et al., 2011; Davidson, 2013; Yadav et al., 2013a, 2013b; Davidson, 2014; Dharmaraj et al., 2014; Garcia et al., 2014; Laneuville et al., 2014; Schrinner et al., 2014; Tilgner, 2014; Christensen, 2015), and the exoplanetary science literature (e.g., Gaidos et al., 2010;

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http://dx.doi.org/10.1016/j.epsl.2015.12.004
0012-821X/

Driscoll and Olson, 2011; Summeren et al., 2013; Zuluaga et al., 2013).

However, in contrast to the extreme parameters which must be reached to observe the asymptotic behavior in other convection systems, dynamo models are presently restricted to rather moderate parameter ranges. It may be the case that the addition of rotational forces and magnetic fields allow asymptotic behaviors to be reached at less extreme parameters. However, these predictions must be verified before claims of achieving an ultimate dynamo physics regime can be made (e.g., Julien et al., 2012b; King and Buffett, 2013).

Stelzer and Jackson (2013) demonstrate that a statistically-significant improvement can be made to Christensen and Aubert (2006)'s scaling laws for flow velocity and magnetic field strength by reintroducing a dependence on viscous and magnetic diffusivities. Thus, they show that diffusion-free empirical fits do not optimally describe flow and magnetic field outputs from present-day dynamo models. Stelzer and Jackson show this via a leave-one-out cross-validation (LOOCV) method, applied to 116 dynamo cases from Christensen and Aubert (2006)'s and following papers. In the LOOCV method, a single case is isolated at a time and a scaling law is constructed from the remaining cases, assuming a power-law relationship between relevant governing parameters. The scaling law is evaluated for its ability to predict the isolated case, and the process is repeated for each case. Using this statistical method, Stelzer and Jackson (2013)'s results make clear that flow and magnetic fields in present-day dynamo models are not yet following a diffusion-free physics, as may be the case in extreme geophysical and astrophysical turbulent flows.

Unlike all other tested quantities, Stelzer and Jackson (2013) find that the heat transfer data are indeed best fit by a diffusion-free scaling relation. Since heat transfer is a globally integrated descriptor of the convection dynamics (e.g., Glazier, 1999), it can be argued that the heat transfer in dynamo models is the first quantity to develop a diffusion-free behavior. It could then be argued that since the heat transfer is following diffusion-free physics, the other quantities will eventually do so as well.

In this paper, however, we show that the goodness of fit of heat transfer data to the scaling proposed in Christensen and Aubert (2006) is determined *a priori* by the way the diffusion-free parameters are defined, rather than by the underlying dynamo physics. In particular, we show that the heat transfer scaling is determined by the onset of convection, which is, in turn, determined by the viscous diffusivity of the fluid. We therefore demonstrate that the scaling is not diffusion-free, contrary to the preconditions for diffusion-free parameters to be applicable.

In Section 2, we define the diffusive and diffusion-free non-dimensional parameters relevant to heat transfer. In Section 3, we introduce the concepts of flattening and shingling that cause the heat transfer data to conform to the onset scaling. We illustrate these concepts using synthetic heat transfer datasets that resemble real dynamo model data. In Section 4, we demonstrate that these concepts lead to a loss of scaling information when real dynamo model data are plotted in diffusion-free parameter space. Regardless of variations in the scaling data, the best-fit scaling that emerges in diffusion-free parameter space is the same. Our results show that data from present-day dynamo models do not provide support for asymptotically-accurate diffusion-free heat transfer physics. Instead, the best fit trend is controlled by the onset of convection in these experiments and has little to do with the bulk turbulent dynamics expected to underly diffusion-free systems (e.g., Julien et al., 2012a).

2. Heat transfer parametrization

2.1. Diffusive parameters

In core dynamics, flows are traditionally described by the magnetohydrodynamic equations that are non-dimensionalized by the relevant time-scales in the problem (e.g., the diffusion times, convection time scale, rotation time, and magnetic induction time scale). This operation generates a (non-unique) set of non-dimensional numbers that are ratios of these various characteristic time scales. Scaling laws associated with heat transfer depend on the Rayleigh number Ra, the Nusselt number Nu, the Prandtl and magnetic Prandtl numbers Pr and Pm, as well as the Ekman number E (e.g., Incropera and DeWitt, 1985; Gubbins and Roberts, 1987; Bergman et al., 2011).

The definitions of Ra, Pr, Pm and E depend on diffusive time scales. Viscous and thermal diffusion time scales are defined respectively as:

$$\tau_\nu = L^2/\nu\,, \quad \tau_\kappa = L^2/\kappa\,, \tag{1}$$

where L is a characteristic length, ν is the viscous diffusivity, and κ is the thermal diffusivity. The time scale for buoyancy forcing τ_{ff} is associated with the convective free-fall velocity of a fluid parcel U_{ff}. This is derived by balancing the inertial term with thermal buoyancy force:

$$\vec{u}\cdot\vec{\nabla}\vec{u} \sim \alpha_T \Delta T \vec{g} \quad \rightarrow \quad U_{ff}(U_{ff}/L) \sim \alpha_T \Delta T g$$
$$\rightarrow \quad U_{ff} = (\alpha_T g \Delta T L)^{1/2}\,.$$

Thus,

$$\tau_{ff} = \frac{L}{U_{ff}} = \left(\frac{L}{\alpha_T g \Delta T}\right)^{1/2}, \tag{2}$$

where α_T is the thermal expansivity coefficient, g is gravitational acceleration and ΔT is the adverse temperature gradient in the fluid.

The Rayleigh number compares the time scales of viscous and thermal diffusivities to that of the free-fall buoyancy time scale:

$$Ra = \frac{\tau_\nu}{\tau_{ff}}\frac{\tau_\kappa}{\tau_{ff}} = \frac{\alpha_T g \Delta T L^3}{\nu\kappa}\,. \tag{3}$$

Rayleigh–Bénard convection (non-rotating, non-magnetic) onsets after a critical value of $Ra \sim 10^3$ (Chandrasekhar, 1961).

The Prandtl number is defined by the ratio between thermal diffusion and viscous diffusion time scales, and is given by:

$$Pr = \frac{\tau_\kappa}{\tau_\nu} = \frac{\nu}{\kappa}\,. \tag{4}$$

While core dynamo regions are estimated to have Pr values between 10^{-1} and 10^{-2} (Pozzo et al., 2012; Davies et al., 2015; Zhang et al., 2015), the majority of dynamo models are carried out at $Pr = 1$ due to computational constraints.

The magnetic Prandtl number is defined by the ratio between magnetic diffusion and viscous diffusion time scales, and is given by:

$$Pm = \frac{\tau_\eta}{\tau_\nu} = \frac{\nu}{\eta} = \mu_0 \nu\sigma\,, \tag{5}$$

where $\tau_\eta = L^2/\eta$; $\eta = (\mu_0\sigma)^{-1}$ is the magnetic diffusivity, σ is the fluid's electrical conductivity and μ_0 is the permeability of free space. Planetary dynamo generating regions are estimated to have Pm values of order 10^{-6} to 10^{-8} (Schubert and Soderlund, 2011). In contrast, for the fluid to be sufficiently inductive to produce dynamo action, the majority of present-day dynamo models must be carried out at $Pm \simeq 1$ (Aurnou et al., 2015).