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Seasonal changes in ice sheet motion due to melt water lubrication

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ABSTRACT

A numerical model is used to calculate how the motion of an idealized ice-sheet margin is affected by the subglacial drainage of melt water from its surface. The model describes the evolution of the drainage system and its coupling with ice flow through a sliding law that depends on the effective pressure. The results predict ice acceleration during early summer when the inefficient drainage system is temporarily overwhelmed. The growth of a more efficient drainage system leads to a subsequent slowdown of the ice very close to the margin, but high water pressure and ice velocity are maintained through much of the summer further inland. Annual mean ice velocity increases with the total quantity of melt water except close to the margin, where it is almost insensitive to the amount of melting. Short-term variability of melt water input leads to rapid changes in ice velocity that result in a slight increase in the mean velocity relative to a smoother input. Linked-cavity and poroelastic models for the distributed drainage system are compared, and their relative merits discussed. Two different sliding laws are considered, and the need for a holistic description of hydraulically controlled sliding is highlighted.

1. Introduction

Much interest has been generated recently in changes of ice motion at the margins of the Greenland ice sheet and the implications these have for its current and future mass balance. Significant variability in ice flow has been observed on time scales of hours to years (Rignot and Kanagaratnam, 2006; van de Wal et al., 2008; Das et al., 2008). Whilst it is not known how new such variability is, the implication is that the ice sheet can react rapidly to oceanic and climatic conditions.

In the case of ocean-terminating outlet glaciers, changes in ice speed are often attributed to changing conditions at the calving front (e.g. Joughin et al., 2008). In other areas, the speed of the ice is clearly influenced by lubrication at its base due to summer melt water descending from the surface (Shepherd et al., 2009; Hoffman et al., 2011). The purpose of this paper is to explore a model of the latter effect, providing a reference with which to interpret the current observations.

Following the established paradigm of alpine glaciers, the structure of the drainage system beneath the ice sheet margins is believed to evolve continually through the seasons (Bartholomew et al., 2010; Sole et al., 2011; Sundal et al., 2011). During winter, with no input from the surface, drainage pathways are squeezed closed and there is a poorly connected system with low transmissivity. When surface melt water reaches the bed in summer, this gives rise to high water pressures that have the effect of reducing basal resistance and increasing ice flow velocities. On the other hand, an increase in the capacity of the drainage system, aided by the melting of well-connected channels, may subsequently reduce water pressures and allow for slower ice velocities even with a larger quantity of melt water. At least, this is the general interpretation. In any case, it is uncertain how much farther (or less) a given section of ice will move in a year with, say, 50% more surface melting.

Numerical models provide a way to address that question, but most ice sheet models have included the effect of basal hydrology in only a rudimentary fashion, if at all. Those that have done so generally incorporate a water layer with a thickness that is directly related to water pressure. The basal shear stress resisting ice flow is then controlled by either the water pressure or the thickness of the water layer (e.g. Alley, 1996). The main application has been to study the evolution of ice streams, where basal melting or freeze-on control the thickness of the water layer. Transport of water is either ignored (Bougamont et al., 2011) or is accounted for by diffusion (Bueler and Brown, 2008; Sayag and Tziperman, 2008; van Pelt and Oerlemans, 2012), or by sheet flow driven by gradients of the ice pressure and basal topography (Johnson and Fastook, 2002; Le Brocq et al., 2009).

More sophisticated models have been proposed to describe the drainage system when a larger quantity of melt water is sourced from the ice surface (Flowers and Clarke, 2002; Creyts and Schoof, 2009; Schoof, 2010). Building on earlier concepts developed for alpine glaciers these models account for the evolving

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transmissivity of the drainage system. The only studies to combine them with ice dynamics are by Pimentel et al. (2010) and Pimentel and Flowers (2011), who demonstrate seasonal evolution of the drainage system and ice velocity in a flow-band model.

In this paper I combine two-dimensional plan-form models of the subglacial drainage system and ice dynamics. I apply the models to an idealized ice sheet margin with the goal to assess, for a simple generic case, what effect summer surface melting has on the speed of the ice. It would be a mistake to base quantitative predictions on these results given the continued uncertainty in some of the parameters; the intention is rather to explore the patterns of change that result.

The model necessarily contains a certain degree of complexity, but to interpret the results it is helpful to understand what is and is not included. The next section therefore explains the model as succinctly as possible, with more technical details included in the Supplementary material. Results of the calculations are shown in Section 3, and in Section 4 I discuss how robust these calculations are and how they compare to observations.

2. Model

2.1. Setup

The ice sheet considered in this paper is shown in Fig. 1. Its size, and the physical parameters used, are intended to be roughly appropriate for the Greenland margins. Even when the ice is moving quickly, the shape of the ice sheet on this scale will change only a small amount over the course of a year. The geometry is therefore treated as fixed. Ultimately, of course, one is interested in the longer term problem of how the ice sheet's motion alters its shape and size.

The model comprises two distinct components—a model for ice flow and a model for subglacial water flow, the base of the ice being assumed to be everywhere at the melting point. Coupling occurs through the basal friction law, which depends upon the subglacial water pressure, and through two feedbacks of the ice flow on the drainage system; namely, the opening of water-filled cavities and the melting caused by the frictional heating of the ice (the latter, as it turns out, is relatively unimportant for this study).

The model is forced by a prescribed rate of water input from the ice surface, r, which varies according to an idealized seasonal cycle as shown in Fig. 1b. This represents all surface melt water and precipitation that is routed to the glacier bed (it may be distinct from the surface melt rate since no account is made of the supra- and en-glacial processes that convert the melting signal to the subglacial input; notwithstanding that, I often refer to r as the melting rate). This inflow is routed to the subglacial drainage system through 50 moulins, the positions of which are randomly prescribed. The input to each moulin is taken from a surrounding catchment basin, A_m, based on a tessellation of the surface, i.e. the input is $R = \int_{A_m} r(s(x,y),t)\,dx\,dy$.

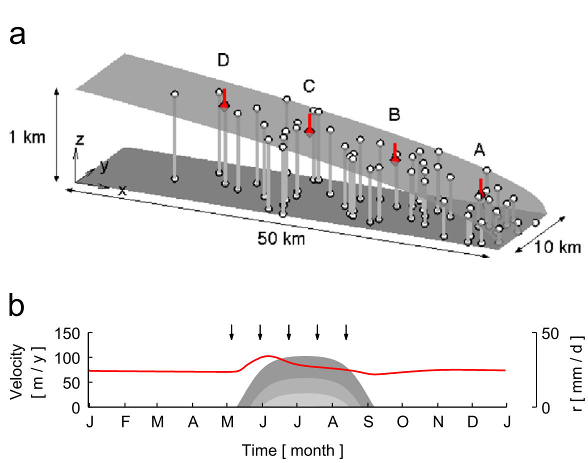

Fig. 1. (a) The ice sheet margin considered in this paper. Dots indicate the position of moulins that route water vertically to the bed; average moulin density is prescribed to increase with proximity to the margin. The bed is flat, $z=b=0$, and the surface is at $z=s=H_0\sqrt{1-x/x_0}$, where $H_0=1060$ m and $x_0=50$ km. (b) Example calculation of ice surface velocity at the position labelled A (red) in response to the idealized seasonal input r (shaded; darkest shading is input at A, lighter shadings show corresponding input at B and C). The input decreases with elevation at lapse rate $r_s=60$ mm d^{-1} km^{-1}, and is given by $r(s,t)=\max\{0,(r_m+r_s s_m)[\frac{1}{2}\tanh(t-t_{spr})/\Delta t-\frac{1}{2}\tanh(t-t_{aut})/\Delta t]-r_s s\}$, where $t_{spr}=$ 135 d, $t_{aut}=244$ d, $\Delta t=21$ d and $s_m=500$ m. The summer peak, and hence the spatial extent of melting, is controlled by the single parameter r_m, which corresponds to the peak rate at 500 m elevation. (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this article.)

2.2. Ice flow

The model for ice flow is a vertically integrated approximation to the Stokes equations. It can be viewed as a combination of the 'shallow ice approximation' and 'membrane stress approximation', and is outlined in the Supplementary material (see also Schoof and Hindmarsh, 2010). It includes longitudinal and transverse stresses that are generated by variable slip at the bed and is therefore most suited to cases of relatively rapid sliding. Importantly, the driving stress due to the ice surface slope can be balanced by these additional stresses as well as by the basal shear stress.

Given the fixed ice geometry, described by basal elevation $z=b(x,y)$ and surface elevation $z=s(x,y)$, the model computes the horizontal velocity components $u(x,y,z)$ and $v(x,y,z)$ resulting from a basal friction law of the form

$$\boldsymbol{\tau}_b = f(U_b, N_+)\frac{\mathbf{u}_b}{U_b}, \tag{1}$$

where $\boldsymbol{\tau}_b$ is the basal shear stress, $\mathbf{u}_b=(u_b,v_b)\equiv(u(x,y,b),v(x,y,b))$ is the sliding velocity, and $U_b\equiv|\mathbf{u}_b|$ is the sliding speed. The function f includes a dependence on the effective pressure, $N=p_i-p_w$, defined as the difference between the average ice normal stress, p_i, and the water pressure in the subglacial drainage system, p_w. In this ice flow approximation the ice normal stress is always hydrostatic, $p_i=\rho_i gH$, where $H(x,y)$ is the ice thickness, ρ_i is the ice density, and g is the gravitational acceleration. Negative effective pressures are treated as zero, so the sliding law more specifically depends on $N_+\equiv\max(N,0)$.

The function f is a pivotal ingredient—it provides the means by which the hydraulic system influences ice motion. For this study I work with the assumption that this coupling occurs through N alone. Specifically, some of the calculations employ a power law of the form

$$f(U_b,N)=\mu_a N^p U_b^q, \tag{2a}$$

where μ_a is a constant parameter and p and q are positive exponents, whilst some use a second law of the form

$$f(U_b,N)=\mu_b N\left(\frac{U_b}{U_b+\lambda_b A N^n}\right)^{1/n}, \tag{2b}$$

where A and n are the coefficients in Glen's law (see Supplementary material), λ_b is a bed roughness length, and μ_b is a limiting roughness slope.

The form in (2a) has some empirical validation (Budd et al., 1979; Bindschadler, 1983), and has been used in a number of numerical ice sheet models (with different exponents; Fowler and Johnson, 1996; Alley, 1996; Bueler and Brown, 2008; van Pelt and Oerlemans, 2012; Bougamont et al., 2011). The form in (2b) is an attempt to capture the effect of cavitation (Fowler, 1986; Schoof, 2005), with a shift from the non-linear viscous drag