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Letters

Cyclic extrusion of a lava dome based on a stick-slip mechanism

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ABSTRACT

Lava dome eruptions are sometimes characterised by large periodic fluctuations in extrusion rate over periods of hours that may be accompanied by Vulcanian explosions and pyroclastic flows. We consider a simple system of nonlinear equations describing a 1D flow of lava extrusion through a deep elastic dyke feeding a shallower cylindrical conduit in order to simulate this short-period cyclicity. Stick-slip conditions depending on a critical shear stress are assumed at the wall boundary of the cylindrical conduit. By analogy with the behaviour of industrial polymers in a plastic extruder, the elastic dyke acts like a barrel and the shallower cylindrical portion of the conduit as a die for the flow of magma acting as a polymer. When we applied the model to the Soufrière Hills Volcano, Montserrat, for which the key parameters have been evaluated from previous studies, cyclic extrusions with periods from 3 to 30 h were readily simulated, matching observations. The model also reproduces the reduced period of cycles observed when a major unloading event occurs due to lava dome collapse.

1. Introduction

In flows of viscous fluids over solid surfaces, slip at the fluid–solid interface is a widely accepted boundary condition experimentally verified by direct observations. Materials like polymers having entangled chains tend to show either wall slip or bulk shear banding depending on the shear rate and chain relaxation time (e.g., Wang et al., 2011). Silicate melts also show complex 3D structures associated with different degrees of polymerisation (Mysen et al., 1982) and here we infer that these structures play a role in magma flow dynamics. The flow of magma in a volcanic conduit, however, is an example with no direct observations of the interface and we must infer the flow conditions there.

In the literature, wall slip is divided into 'true slip' where there is a discontinuity in the velocity field at the fluid–solid interface, and 'apparent slip' where there is an inhomogeneous thin layer of fluid adjacent to the wall with different rheological properties to the bulk of fluid which promotes fluid motion (see Sochi, 2011 for an extensive review). For the case of the 'apparent slip' a large velocity gradient across a very thin low-viscosity slip layer occurs, effectively producing slip at the wall although strictly the non-slip condition is not violated (e.g., Kalyon, 2005). The apparent slip becomes more pronounced as the viscosity of the slip layer decreases relative to the viscosity of the bulk of the fluid. Often slip appears together with, or as consequence of, other overlapping effects, such as shear banding and viscous heating. In most cases apparent slip, rather than true slip, is the more common and viable mechanism for observable wall slip (Sochi, 2011).

A stick-slip mechanism was proposed as an explanation for cyclic behaviour observed during the extrusive flow of magma from silicic volcanoes (e.g., Denlinger and Hoblitt, 1999; Iverson et al., 2006; Lensky et al., 2008). Denlinger and Hoblitt (1999) heuristically suggested that cyclicity results from a flow of compressible magma through a volcanic conduit combined with a stick-slip condition along the wall of the upper conduit, similar to the behaviour of the extrusion of industrial polymers (Den Doelder et al. 1998). Stick-slip behaviour of a solid plug at the top of the volcanic conduit has also been proposed to explain some observed cyclic activity at volcanoes. At Mount St. Helens (MSH), USA, Iverson et al. (2006) described periodic seismicity as a consequence of stick-slip flow of a solid magma plug pushed by compressible magma. They reproduced oscillations on a timescale of minutes, proposing a dynamic model that had behaviour similar to that of a variably damped oscillator. Stick-slip transition in andesitic magma near the conduit wall was also invoked as an explanation for the flow cycle prior to the Strombolian eruptions of Karymsky Volcano, Russia (Ozerov et al., 2003). Lensky et al. (2008) presented a model of cyclic volcanic activity at the Soufrière Hills Volcano (SHV), Montserrat, coupling magma flow with degassing of supersaturated magma, gas escape from

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the permeable magma, deformation of the conduit walls and the friction between the walls and the plug at the top of the conduit.

Experiments on high viscosity crystallised andesitic magmas, typical for lava dome eruptions, showed the occurrence of frictional melting (Lavallée et al., 2012). In these experiments slip appeared at a shear stress of about 1.5–2 MPa. They identified a very thin layer (~0.4 mm) of chemically homogeneous frictional melt containing resorbed crystals where the viscosity is reduced by six orders of magnitude. In the upper part of a lava dome, where the viscosity is extremely large due to a high degree of microlite crystallisation, frictional melting can be a viable mechanism for promoting wall slip. Shear bands in a volcanic conduit were modelled by Hale and Mühlhaus (2007) for a high crystallinity magma similar to that of the SHV, Montserrat. Shear bands were simulated using a strain-localisation model when the shear stress equals the magma shear strength. However they did not model friction-controlled slip but only a plastic flow. They showed that shear bands can form to depths of a few hundreds metres. It is worth noting that, in order to obtain shear band depths of about 700 m in their analysis they had to assume low magma shear strengths (2×10^5 Pa) but they neglected completely any shear heating effects. Results from frictional melt experiments (Lavallée et al., 2012) and computational results for viscous magma flows (Costa and Macedonio, 2003; 2005; Costa et al., 2007a; Hale et al., 2007) indicate that in conditions typical for lava dome flows, viscous heating has a major contribution to the formation of effective shear bands.

Here, we generalise the heuristic model proposed by Denliger and Hoblit (1999) for SHV and make it suitable for the combined dyke-cylinder geometry of Costa et al. (2007b). We use a quantitative slip relationship similar to the slip law inferred for polymer flow, and constrain all physical parameters necessary to the model with the most recent estimation derived from both observations and models, and a combination of both (e.g., Costa et al., 2007b; Odbert and Wadge, 2009). In particular we develop a general framework based on a simplified quantitative model for lava dome extrusion that, invoking wall–slip conditions and a partially elastic conduit, is able to explain the sub-daily cyclic behaviour of magma flow and lava extrusion, observed at the SHV, with periods from about 3–30 h, with a mean of ~9 h (Voight et al., 1999).

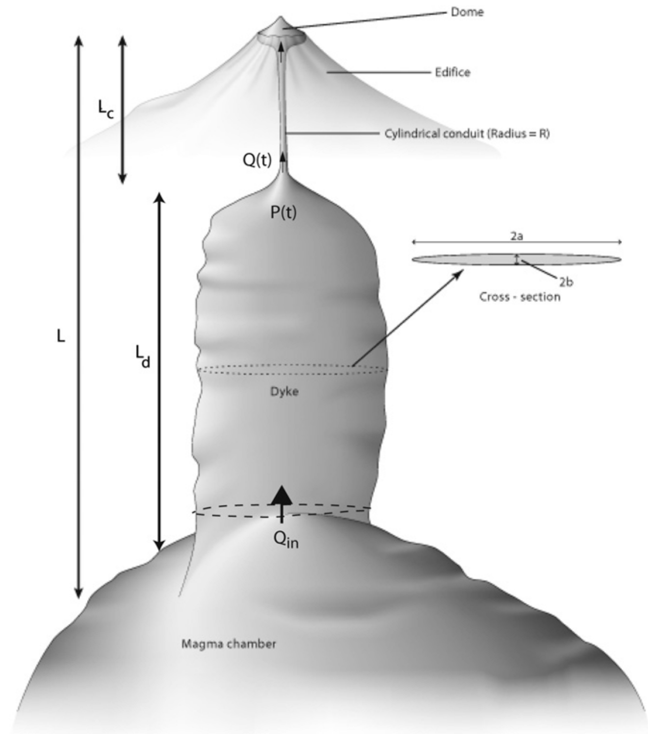

Fig. 1. Simplified sketch of the investigated system. Modified after Costa et al. (2007c).

2. Mathematical model of lava extrusion

Let us consider a simple system consisting of an elastic dyke of height L_d, and width W_d (where $W_d=2a$) that feeds a cylindrical conduit of length L_c and radius R and is filled with magma (Fig. 1). This system is similar to an industrial polymer extruder, where the dyke acts as the barrel, the conduit as the die and the magma as a polymer.

We assume that the relationship between the pressure at the top of the dyke P and the mass influx Q_{in} and mass outflux Q_{out} of magma to and from the dyke is given by

$$\frac{dP}{dt}=\frac{\gamma}{\rho V_d}(Q_{in}-Q) \tag{1}$$

where $Q(t)=Q_{out}=2\pi\,\rho_d\int_0^R v(r,t)rdr$ (with t denoting time, r radial coordinate, and v magma velocity), V_d is the dyke volume, ρ is the average magma density, and γ is an effective compressibility–rigidity modulus that, in principle, can be expressed as a function of the magma bulk modulus, K, and the rigidity modulus of rocks surrounding the dyke, G.

Costa et al. (2007b,c) developed a model for magma flow feeding lava dome extrusion whose components comprised a dyke below a cylindrical conduit. The model accounts for degassing-induced crystallisation kinetics, gas exsolution and filtration through the magma, rheological stiffening of magma due to crystallisation, and latent heat release (Melnik and Sparks, 1999, 2005). As it considers a dyke geometry, Costa et al. (2007b,c) accounted for variations in conduit cross-section due to elastic deformation of the wallrocks. Rather than choose the value of γ arbitrarily or from a simplified zero dimensional physical model, we estimate its typical range of values using the geometry and the physical parameters given by the Costa et al. (2007b) model for the SHV dyke-cylinder system. Using this approach (explained in detail in the Appendix A) we found that γ is not constant but varies on a time scale of the order of weeks (Fig. 2). The variation of the compressibility–rigidity modulus, γ, is controlled by the compressibility of the magma and elastic expansion of the dyke. These factors lead to changes of the effective compressibility–rigidity modulus. However, the nonlinear interplay between the pressure variation rate dP/dt and the magma budget $(Q-Q_{in})$ leads to a more complicated variation of the modulus. Since here we aim to capture the first-order behaviour of the system, we explore the variability of γ in a parametric way and use the results from the Costa et al. (2007b) model to estimate the typical range of values of γ. We focus in this paper on processes with a timescale of the order of hours, whereas γ fluctuates on timescale of weeks, so a quasi-steady approach can be used and γ can be treated as a constant in the range from 0.01 GPa to 5 GPa (see Fig. 2 and Appendix A). A more general model that explicitly describes γ as a time-dependent parameter is the subject of further work.

The momentum equation for flow of magma with viscosity μ inside the cylindrical conduit is:

$$\rho\frac{\partial v}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)=-\frac{\partial P}{\partial z}-\rho g\equiv-\frac{\partial\tilde{P}}{\partial z} \tag{2}$$

where z denotes the vertical coordinate and g gravitational acceleration. As a first approximation we can neglect fluid compressibility and wallrock elasticity in the cylindrical conduit. Moreover, since the Reynolds number, $\mathrm{Re}=\rho VR/\mu$, is typically very small,