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A phase-space model for Pleistocene ice volume

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ABSTRACT

We present a phase-space model that simulates Pleistocene ice volume changes based on Earth's orbital parameters. Terminations in the model are triggered by a combination of ice volume and orbital forcing and agree well with age estimates for Late Pleistocene terminations. The average phase at which model terminations begin is approximately $90 \pm 90^{\circ}$ before the maxima in all three orbital cycles. The large variability in phase is likely caused by interactions between the three cycles and ice volume. Unlike previous ice volume models, this model produces an orbitally driven increase in 100-kyr power during the mid-Pleistocene transition without any change in model parameters. This supports the hypothesis that Pleistocene variations in the 100-kyr power of glacial cycles could be caused, at least in part, by changes in Earth's orbital parameters, such as amplitude modulation of the 100-kyr eccentricity cycle, rather than changes within the climate system.

1. Introduction

Numerous studies have demonstrated that Pleistocene glacial cycles are linked to cyclic changes in Earth's orbital parameters (Hays et al., 1976; Imbrie et al., 1992; Lisiecki and Raymo, 2007); however, many questions remain about how orbital cycles in insolation produce the observed climate response. The most contentious problem is why late Pleistocene climate records are dominated by 100-kyr cyclicity. Insolation changes are dominated by 41-kyr obliquity and 23-kyr precession cycles whereas the 100-kyr eccentricity cycle produces negligible 100-kyr power in seasonal or mean annual insolation. Thus, various studies have proposed that 100-kyr glacial cycles are a response to the eccentricity-driven modulation of precession (Raymo, 1997; Lisiecki, 2010b), bundling of obliquity cycles (Huybers and Wunsch, 2005; Liu et al., 2008), and/or internal oscillations (Saltzman et al., 1984; Gildor and Tziperman, 2000; Toggweiler, 2008).

A closely related problem is the question of why the dominant glacial cycle shifted from 41 kyr to 100 kyr during the mid-Pleistocene. Most commonly, this mid-Pleistocene transition (MPT) is attributed either to a drop in atmospheric CO_2 levels (Raymo, 1997; Paillard, 1998; Hönisch et al., 2009) or erosion of the continental regolith (Clark et al., 2006), but other mechanisms have also been proposed. Some hypotheses do not require any internal changes in the climate system, attributing the shift to chaotic or irregular mode-switching (Saltzman and Verbitsky, 1993; Huybers, 2009), or to a change in the character of insolation forcing (Lisiecki, 2010b). For example, the 100-kyr power of the climate response is observed to be negatively correlated with the 100-kyr power of eccentricity for at least the last 3 Myr (Lisiecki, 2010b; Meyers and Hinnov, 2010).

Many simple models have produced 100-kyr cycles as responses to precession and/or obliquity using different nonlinear responses to orbital forcing. Early models had difficulty reproducing the large amplitude of Marine Isotope Stage (MIS) 11 during weak orbital forcing and produced too much 400-kyr power (e.g. Imbrie and Imbrie, 1980). However, newer multi-state models have solved these particular problems (Paillard, 1998; Parrenin and Paillard, 2003). Many models can also reproduce a transition from 41-kyr to 100-kyr cyclicity during the mid-Pleistocene transition by changing certain model parameters or climate boundary conditions (Raymo, 1997; Paillard, 1998; Ashkenazy and Tziperman, 2004; Paillard and Parrenin, 2004; Clark et al., 2006; Huybers, 2007). In fact, the wide variety of ways in which 100-kyr glacial cycles can be produced makes it difficult to determine which, if any, of the models correctly describes the source of 100-kyr glacial cyclicity (Tziperman et al., 2006).

We present a new, phase-space model of Pleistocene ice volume that generates 100-kyr cycles in the Late Pleistocene as a response to obliquity and precession forcing. Like Parrenin and Paillard, (2003), we use a threshold for glacial terminations. However, ours is a phase-space threshold: a function of ice volume and its rate of change. Our model is

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the first to produce an orbitally driven increase in 100-kyr power during the mid-Pleistocene transition without any change in model parameters. In section **2**, we describe the model and the derivation of its parameters. In section **3**, we compare the model results and climate data for the last 3 Myr, with emphasis on the timing of 100-kyr glacial terminations and changes in spectral power. In section **4**, we discuss (1) parameterization of the relationships between ice volume and orbital forcing, (2) the timing of terminations with respect to orbital forcing, and (3) the mid-Pleistocene transition. Finally, section **5** summarizes our conclusions.

2. Methods

2.1. Overview

We use a statistical analysis of the ice-volume record to guide the development of a set of evolution equations which accurately model the dynamics of glacial cycles. This is an important shift in perspective, in that we are not testing specific physical mechanisms that may be responsible for key features of the record. Rather, we hope that the form of the equations will help clarify the discussion of possible mechanisms. Assuming that ice volume is the slowest mode in the climate system, we look for equations involving a single variable y and an orbital forcing term, and then explain as much of the low-frequency variation in the record as possible.

2.2. Variables

Development and parameterization of the model is guided by analysis of the LR04 global stack of benthic δ^{18}O (Lisiecki and Raymo, 2005), which is a proxy for global ice volume and deepwater temperature. The LR04 stack from 0 to 1500 ka is taken with a sign change, as is conventional so that larger values correspond to warmer epochs (smaller ice volume). The stack is interpolated as needed to get a record sampled every thousand years. As we are interested in the slowly varying aspects of the record, we put the data through a Gaussian notch filter centered at 0 with a bandwidth of .1 kyr^{-1}. The result was standardized by subtracting the mean and dividing by the standard deviation, producing a function $y(t)$ for the ice-volume record over time scaled to run roughly from -2 to 2.

Combinations of orbital functions ε (obliquity), $e \sin \omega$ (precession) and $e \cos \omega$ (phase-shifted precession) are used as a forcing for our model, and are taken from (Laskar et al., 2004). Insolation at most latitudes and seasons can be represented quite accurately by a combination of these three orbital functions. The variables were standardized based on the mean and standard deviation from 0 to 1500 ka, with a thousand year sampling interval. The resulting variables are denoted *obl*, *esi*, *eco*.

2.3. Phase-space picture

We examine the ice-volume record in a 2-dimensional plot, with y on the horizontal axis and y' (the time rate of change of y) on the vertical axis. Positive values denote $y'>0$ (warming epochs). Colors denote the forcing function (discussed below), with red for large positive (warming) and blue for large negative (cooling). One thing that becomes clear in Fig. 1 is the special role that the large semicircular loops play in the ice-volume dynamics. These are traversed in a clockwise fashion and correspond to periods of rapid warming (terminations). Terminations have long been recognized as key features of the ice-volume record. Most of the time, however, the system remains fairly close to the horizontal axis, moving gradually to the left and bobbing up and down as it adapts slowly to changes in forcing.

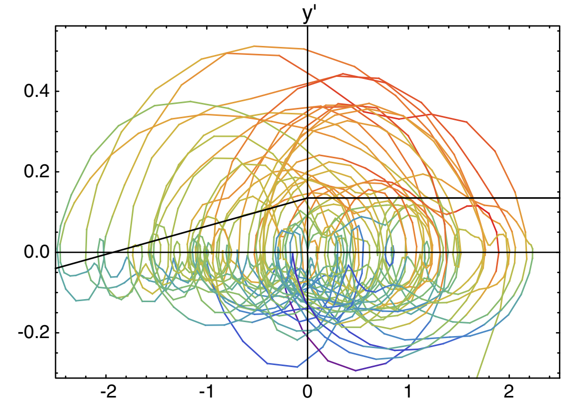

Fig. 1. Phase-space trace of the filtered ice-volume record. Colors indicate forcing. Black line indicates the function *trans*(x) which delineates the model's boundary between the accumulation state and the termination state (See supplementary online video).

2.4. Threshold for terminations

In Fig. 1 one can observe just above the horizontal axis that there is a transition zone where some loops head back to the axis and others head upwards to initiate a large loop. Evidently, if the rate of warming is large enough, the climate will shift into a termination mode with runaway melting. The threshold appears to be a diagonal line extending roughly from $(-2, 0)$ to $(0, .13)$. This leads us to define a transition line

$$trans(x) = \; min\{.135 + .07x, .135\} \tag{1}$$

(plotted onto Fig. 1) which incorporates this diagonal line and extends it horizontally to the right of the vertical axis. Above the line, we consider the system to be in termination state; below it is in glaciating state. Multi-state models have been employed with considerable success by (Parrenin and Paillard, 2003) using a threshold which depends on a combination of insolation and ice volume. Here the diagonal line represents a combination of y' and y (ice volume) — but insofar as y' and insolation are correlated, the concepts are similar.

2.5. Accumulation state

The accumulation state is modeled with a first-order differential equation

$$y' = g(y) + F(y,t), \tag{2}$$

where $g(y)$ describes the internal tendency of the ice sheets to grow or retreat, and $F(y,t)$ describes the external forcing. A first-order equation is a natural starting point for a system responding to a variable heat source. Additionally, the phase lag with respect to obliquity is characteristic of a first-order equation. As we will see, there is significant variation in y of the sensitivity of ice volume to various components of orbital forcing. This is the reason for allowing the forcing function to depend on y as well as t.

Clues for what might be reasonable choices for g were obtained by a weighted regression analysis. For the period 1500–0 ka, a weighted least squares regression was performed for y' with *obl*, *esi*, *eco* used as explanatory variables. The following weights were used: $exp[-(y(t)-y_0)^2/d^2]$, with $d=.45$. In this way, we were able to get an understanding of the contributions of the individual forcing functions to y'. Furthermore, we could learn how the contributions vary with ice volume y_0 by