Earth and Planetary Science Letters 304 (2011) 45–54

Contents lists available at ScienceDirect

Earth and Planetary Science Letters

journal homepage: www.elsevier.com/locate/epsl

A simple analytical solution for slab detachment

Stefan M. Schmalholz *

Institute of Geology and Palaeontology, University of Lausanne, 1015 Lausanne, Switzerland

A R T I C L E   I N F O

Article history:
Received 9 June 2010
Received in revised form 9 January 2011
Accepted 11 January 2011
Available online 22 February 2011

Editor: Y. Ricard

Keywords:
necking
slab detachment
strain localization
analytical solution
instability
buoyancy stress

A B S T R A C T

An analytical solution is presented for the nonlinear dynamics of high amplitude necking in a free layer of power-law fluid extended in layer-parallel direction due to buoyancy stress. The solution is one-dimensional (1-D) and contains three dimensionless parameters: the thinning factor (i.e. ratio of current to initial layer thickness), the power-law stress exponent, n, and the ratio of time to the characteristic deformation time of a viscous layer under buoyancy stress, t/t_c. t_c is the ratio of the layer's effective viscosity to the applied buoyancy stress. The value of t_c/n specifies the time for detachment, i.e. the time it takes until the layer thickness has thinned to zero. The first-order accuracy of the 1-D solution is confirmed with 2-D finite element simulations of buoyancy-driven necking in a layer of power-law fluid embedded in a linear or power-law viscous medium. The analytical solution is accurate within a factor about 2 if the effective viscosity ratio between the layer and the medium is larger than about 100 and if the medium is a power-law fluid. The analytical solution is applied to slab detachment using dislocation creep laws for dry and wet olivine. Results show that one of the most important parameters controlling the dynamics of slab detachment is the strength of the slab which strongly depends on temperature and rheological parameters. The fundamental conclusions concerning slab detachment resulting from both the analytical solution and from earlier published thermo-mechanical numerical simulations agree well, indicating the usefulness of the highly simplified analytical solution for better understanding slab detachment. Slab detachment resulting from viscous necking is a combination of inhomogeneous thinning due to varying buoyancy stress within the slab and a necking instability due to the power-law viscous rheology ($n>1$). Application of the analytical solution to the Hindu Kush slab provides no "order-of-magnitude argument" against slab detachment and, therefore, supports existing studies suggesting a currently ongoing slab detachment in the Hindu Kush slab.

1. Introduction

Necking is a mechanical instability that can occur in plastic and viscous layers during layer-parallel extension (e.g. Hill and Hutchinson, 1975; Smith, 1977). Necking is characterized by the localization of strain and thinning in the layer. In rocks, necking is often associated with brittle behavior, tensile failure or normal faulting but diffuse, continuous necking may be a moderate-strain precursor which controls the initial stages of extension and triggers strain localization. Necking occurs from the centimeter to the lithospheric scale and fluid dynamic necking models have been used to explain, for example, small-scale pinch-and-swell structure (e.g. Schmalholz et al., 2008; Smith, 1977) or lithospheric extensional structures (e.g. Ricard and Froidevaux, 1986) such as the Basin-and-Range Province (Fletcher and Hallet, 1983) or rift zones (e.g. Zuber and Parmentier, 1986).

A geodynamic process which is likely also controlled, at least to some extent, by necking is slab detachment (e.g. Andrews and Billen, 2009; Davies and Von Blanckenburg, 1995; Gerya et al., 2004; Wong-A-Ton and Wortel, 1997). During slab detachment the subducted lithospheric plate becomes detached from the surface plate. Slab detachment has been suggested based on gaps in hypocenter distributions and within tomographic images of subducted slabs (e.g. Spakman et al., 1988; Wortel and Spakman, 2000). A number of studies suggest slab detachment as a possible explanation for geological and geophysical observations in many regions worldwide, such as the Tasman Sea (e.g. Schellart et al., 2009), the Carpathians (e.g. Cloetingh et al., 2004; Wortel and Spakman, 2000) or the India-Asia collision zone (e.g. Kosarev et al., 1999; Van der Voo et al., 1999). These observations include the timing of magmatism, volcanism and geochemical analysis (Von Blanckenburg and Davies, 1995), and they suggest that the timing of slab detachment is on the order of a few million years (Andrews and Billen, 2009). Slab detachment has also been studied with laboratory experiments and applied to explain parts of the tectonic evolution of the Himalaya–Tibet system (Chemenda et al., 2000) or the North Anatolian fault system (Faccenna et al., 2006). All these studies suggest that slab detachment is an important geodynamic process that has significant impact on plate motion, lithospheric deformation and the temperature field in the lithosphere and mantle.

However, only few studies have investigated the dynamics and thermo-mechanics of slab detachment theoretically. For example,

* Tel.: +41 21 692 4302; fax: +41 21 692 4305.
E-mail address: stefan.schmalholz@unil.ch.

0012-821X/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.epsl.2011.01.011

Davies and Von Blanckenburg (1995) investigated differences in the numerically integrated strength across continental and oceanic subducting lithospheric slabs. They concluded that slab detachment is theoretically plausible for natural conditions based on their strength and buoyancy calculations. Wong-A-Ton and Wortel (1997) performed a similar strength analysis but used a transient temperature model. They concluded that the strength of the subducting lithosphere, which is controlled by its temperature and rheology, is one of the most important parameters for slab detachment. They also estimated that slab detachment takes place on a time-scale of 1.7 to 39 Myr. These studies provide important results concerning the impact of lithospheric strength on slab detachment. However, both studies used models that are not dynamic. Consequently, these studies had to assume a particular, constant strain rate in the subducting lithosphere and do not provide any insight into the dynamics of slab detachment. Therefore, also fully dynamic two-dimensional (2-D) numerical simulations were performed to study the dynamics of slab detachment (e.g. Andrews and Billen, 2009; Baumann et al., 2010; Burkett and Billen, 2009; Duretz et al., in press; Gerya et al., 2004). These studies also concluded that the rheology and thermal structure of the slab are the first order parameters controlling slab detachment. Gerya et al. (2004) showed that the detachment process can be caused by thermal diffusion and the related heating of subducted slabs after cessation of active subduction. Andrews and Billen (2009) concluded, amongst other things, that slab detachment does not occur for a purely Newtonian rheology. Recently, Burkett and Billen (2010) and Van Hunen and Allen (2010) presented dynamic 3-D numerical simulations to study the lateral propagation of slab detachment. All these numerical models provide important results for a better understanding of the dynamics of slab detachment, but they are already quite complex. Consequently, it is not obvious how to determine specific dimensionless parameters that control slab detachment from the numerical results and how these parameters are related to each other. Therefore, more theoretical work on slab detachment is still needed and justified, especially work on analytical solutions capturing the first-order dynamics and parameters.

In this study, a simple closed-form analytical solution is presented for the necking in a free layer of power-law fluid which is extended due to buoyancy stress acting parallel to the layer (Fig. 1A). The first-order accuracy of the analytical solution is confirmed by numerical finite element simulations of 2-D buoyancy-driven viscous fluid flow for situations where the layer is additionally surrounded by viscous fluids. Although the analytical solution is derived for a highly simplified necking scenario, it is applied to investigate slab detachment. Dislocation creep laws for wet and dry olivine are applied as effective slab rheology. It is shown that most of the general conclusions concerning slab detachment of previously published, above mentioned, numerical studies agree well with conclusions resulting from the analytical solution. The analytical solution is further applied to the Hindu Kush slab and the results support the hypothesis of currently ongoing slab detachment (e.g. Lister et al., 2008).

The aim of this study is, first, to present a simple analytical solution providing the controlling dimensionless parameters and capturing the first-order dynamics of high amplitude viscous necking driven by buoyancy stress and, second, to show that this analytical solution is useful for a better understanding of the complex geodynamic process of slab detachment.

2. Analytical solution

An analytical solution for high amplitude necking of a free (i.e. total stress normal to the layer is zero) layer of power-law fluid under layer-parallel extension can be obtained based on the assumptions that plane sections across the layer remain plane during deformation and that the layer surfaces are traction-free (e.g. Schmalholz et al., 2008). By incompressibility in plane flow, the assumed uniform rate of deformation parallel to the layer, $\dot{\varepsilon}$, of a plane in the layer with local thickness, D, is

$$\dot{\varepsilon} = -\frac{1}{D}\frac{dD}{dt} \quad (1)$$

with d/dt being the total derivative with respect to time. For an incompressible, free layer the layer-parallel force, F, is uniform and the mean layer-parallel deviatoric stress, τ, is half the mean layer-parallel total stress, F/D, (e.g. Turcotte and Schubert, 1982) and is

$$\tau = \frac{1}{2}\frac{F}{D} \quad (2)$$

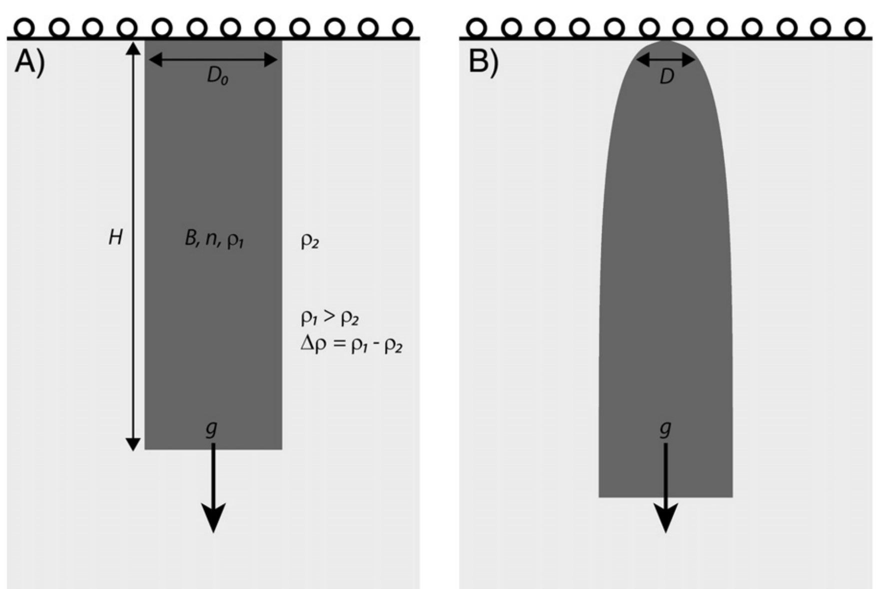

Fig. 1. A) Sketch showing the parameters used for the analytical solution. A layer with length, H, and thickness, D_0, of power-law fluid with stress exponent, n, and coefficient, B, has a larger density, ρ_1, than the inviscid medium with density, ρ_2, surrounding the layer. The layer is attached at its top and gravity acceleration, g, acts downward in a direction parallel to the layer boundaries. B) Typical "cusp" shape of the necked layer with reduced thickness, D, as predicted by the analytical solution.