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Lattice Boltzmann method for groundwater flow in non-orthogonal structured lattices

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ABSTRACT

The efficiency of the lattice Boltzmann method (LBM) in modeling isotropic groundwater flow in domains of arbitrary geometry has been investigated. The Poisson equation was transformed in general curvilinear coordinates. The corresponding equilibrium function for the D2Q9 square lattice based on metric function between the physical and the computational domain has been established. The resulting LBM was checked on examples having higher generality; flows in confined and unconfined aquifers, in vertical and horizontal plane have been considered. In addition, the phreatic water table representing upper boundary in the vertical plane was determined by the dynamic σ-stretching approach, not requiring complex concepts for dealing with the free surface (like the volume of fluid method). The accuracy and stability of the model is controlled by the adaptive mesh concept. This allows application of higher density grid in critical areas with high pressure and velocity gradients, and vice versa. The number of computation points is significantly reduced without loosing accuracy. The basic characteristics of the LBM including features like parallelization and simplicity, are maintained. The advantages of the proposed curvilinear LBM in modeling groundwater flow in domains of complex shape over the former published methods is demonstrated by three examples.

1. Introduction

Based on kinetic structure – where fluid is treated as particles colliding in discretized phase–space system – the LBM represents relatively new numerical approach for solving nonlinear partial differential equations. Basically derived from the concept of Lattice Gas Automata (LGA) [1,2], the LBM is characterized by local nature allowing parallel computation, high accuracy and simplicity in the implementation of the computer code. As a result, in the recent two decades this concept is exploited for investigation of various complex flow related physical processes in the field of civil and environmental engineering. Several papers have been published regarding the application of the LBM to particular domains of interest. However, groundwater flow – described by relatively simpler mathematical formulation in form of second order partial differential equations of parabolic type known as Laplace and Poisson equations – has been investigated just by few authors. The first application of the LBM to pure diffusion was performed by Gladrow [3]. The applicability of the LBM to isotropic groundwater flow in horizontal plane in case of confined aquifer (LABGIF) was investigated by Zhou [4]. In order to reduce limitations imposed by square lattice, further improvement of this approach was realized by the same author by introducing

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the LBM using rectangular lattice [5] for modeling groundwater flow. Application of the LBM to the shallow water equations and Navier–Stokes equations – as well as to transport phenomena related to open channel flow – was investigated by significant number of authors. Background for the development of equilibrium distributions, force terms and test examples in application of the LBM to the shallow water equations was given by Zhou [6]. Detailed investigation of the method and successful application to different test cases was performed by Dellar [7], Liu et al. [8], Thömmes et al. [9], Liu et al. [10] and Linhao et al. [11]. Transport phenomena in case of open channel flow was investigated by Li and Huang [12], Banda et al. [13] and Peng et al. [14], while free surface problems (dam failure, hydraulic jumps) and three-dimensional simulations of turbulent flow can be found in [15–17].

For the most efficient use of the main features of the LBM in solving practical groundwater flow related problems, further adjustments and improvements of the method are required. In general, geometry created by nature is arbitrary and complex. Therefore, at first the basic LBM needs to be adapted to the domain of given arbitrary geometry. Several attempts have been made to adapt the lattice concept to flow domains of curved boundaries. Local hierarchical grid refinement on the classic uniform calculation grid was investigated by Filippova and Hänel [18], while He et al. [19] and Shu et al. [20] utilized the interpolation-supplemented scheme (ISLBE), as well as its version improved by Taylor series expansion and optimization using the least squares method. In order to make the lattice concept more square free, group of authors aimed their research at the modification of the basic form of the equilibrium distribution function. In the single relaxation time form of the lattice Boltzmann method – known as LBGK model (lattice Boltzmann Bhatnagar–Gross–Krook) [21] – Zhou [22] has introduced a new local equilibrium distribution function for non-uniform grids, while Budinski [23] applied curvilinear transformation to the shallow water equations, which was then successfully implemented in the LBGK, not causing any deterioration of the basic structure of the method. In contrast to the LBGK, application of *two-relaxation-time* method (TRT) – known for providing more stable solution – is considered and analyzed by the Ginzburg et al. [24] and Ginzburg [25]. On the other hand, the overall stability of the lattice Boltzmann method is enhanced using the multi relaxation approach (MRT) [26], which was investigated by Bouzidi et al. [27] in case of non-uniform grids. Implementation of the MRT approach in case of complete transformation of shallow water equations in curvilinear coordinates was accomplished by Budinski [28]. Since issues due to the arbitrary geometry of the flow domain emerge in modeling of ground water flow as well, in this paper curvilinear transformation is applied to the basic flow equations in horizontal and vertical plane. Furthermore, as isotropic groundwater flow in horizontal plane is considered both in confined and unconfined aquifer, the corresponding equilibrium distribution functions are developed. Since in case of phreatic aquifer the position of the water table (upper boundary) is not known in advance, modeling in vertical plane is rather complicated. For that the transformed curvilinear equation is combined with the σ−stretching approach, in order to adapt the mesh to the changing position of the water table, representing the upper boundary of the model. The present attempt of incorporating the water table dynamics into the LBM is generally based to the Volume of Fluid method (VoF) and the kinematic boundary condition approach (KBC). Application of the first method is investigated by Janßen and Krafczyk [15], Janßen et al. [29] and Thürey and Rüde [30], while the KBC approach is exploited by Zhao et al. [31]. In this paper the water table is seen as unsteady boundary which changes its position in vertical direction. Vertical tracking of the water surface is achieved using the stretching method, which allows shrinking/expanding of the computational mesh in vertical direction in accordance with the actual position of the free surface representing upper boundary. Three different examples with exact analytical solution, or solution obtained by traditional computational procedures (finite difference method) have been used for verification and validation of the outlined LBM.

2. The mathematical model

2.1. Application of the curvilinear transformation to the Poisson's equation

In order to cover wide spectrum of cases concerning two-dimensional groundwater flow, unsteady isotropic groundwater flow in (x_1, x_2) plane is considered, described by the Poisson equation [32] in its general form,

$$\frac{\partial h}{\partial t} = \frac{T}{S}\frac{\partial^2 \Phi}{\partial x_i^2} + \frac{R}{S}, \quad i = 1, 2, \tag{1}$$

where t is time, T is transmissivity, S is storativity, R is the recharge function and x_i are the Cartesian coordinates. For confined aquifer, transmissivity T is calculated as $T = K \cdot d$, where K is the hydraulic conductivity and d is the saturated thickness of the aquifer, while S is calculated as $S = S_s \cdot d$, where S_s is the specific storage. However, for unconfined aquifer, where flow happens through the entire vertical cross section, saturated thickness d becomes water head, transmissivity T changes to K, while storage coefficient S becomes specific yield S_y. Water head h is formulated according to the type of the aquifer, and it can be defined using the parameter Φ as:

$$\Phi = \begin{cases} h, & \textit{confined} \\ \dfrac{h^2}{2}, & \textit{unconfined}. \end{cases} \tag{2}$$

Since Eq. (1) describes groundwater flow in the orthogonal Cartesian coordinate system, the choice of appropriate numerical grid is conditioned by the shape of the boundaries, which can be approximated by straight, orthogonal solid lines. From an