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On the numerical simulation of the nonbreaking solitary waves run up on sloping beaches

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ABSTRACT

In this paper, a solitary wave propagation problem running up on a range of relatively steep slopes to gentle slopes is investigated. Boussinesq solitary wave generation method is used in five beach slopes with the still water heights of 15.0–68.6 cm and relative wave heights of 0.046–0.348. The incompressible smoothed particle hydrodynamics method is utilized as the numerical method. Simulation results are compared with analytical results and experimental data in terms of free surface displacement, maximum run up, shoreline movement and fluid particle velocity. Surface profile results agree reasonably with the analytical results at run down and horizontal particle velocities coincide with the experimental data. Furthermore, the results of maximum run up show that for moderate and large wave heights, Lagrangian numerical results pursuit the maximum run up of the experimental data with acceptable accuracy.

1. Introduction

The aim of this investigation is the numerical simulation of the run up of solitary waves on plane beaches with various slopes. Of particular importance is evaluating the maximum run up and velocity profiles. Here, nonbreaking water waves are considered. Solitary waves can predict many of the characteristics of shoreward inundation of calamitous waves [1–3]. In order to yield results applicable to three dimensional numerical simulations of coastal sites, a simple two dimensional case of a solitary wave propagating on a constant water depth and impinging on a plane sloping beach can be used. Hence, the important characteristics of the run up phenomena can be obtained using this simplified model. The investigation of the wave generation, wave propagation, breaking, and the run up process has been the subject of numerous experimental [4–9], analytical [1,10–17], and numerical [2,6,18–25] studies in recent decades.

So far, a wide range of numerical methods have been used to simulate run up of solitary waves on the slope. Among them, some researchers were utilized mesh free methods for analyzing the free surface phenomena. Incompressible Smoothed Particle Hydrodynamics (ISPH), as a meshless particle method, may be mentioned as the most famous meshless method to model the free surface flows. Wave generation is held in a numerical two dimensional wave channel by a piston with prescribed motion. The wave maker is located on the left side of the wave channel and by generating solitary wave, it moves from left side toward the shoreline.

In a sufficient propagation domain, Boussinesq solitary wave generation method with relative wave heights of 0.046–0.348 with the still water heights of 15.0–68.6 cm and beach slopes of 1:2.14, 1:2.75, 1:3.73, 1:5.67, and 1:11.43 is investigated using ISPH method and then the results of this solitary wave generation method over the beach slope range

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is analyzed. To do this, numerical free surface elevation of the wave will be compared with analytical results at some times and different distances away from the paddle. Then, the wave peak amplitudes at different locations will be discussed and compared to the linear and nonlinear theories. Finally, the shoreline movement, velocity components distributions and maximum run up of solitary waves will be inspected with respect to the laminar experimental data of [9].

2. Governing equations and prediction–correction scheme

The motion of a continuum in the Lagrangian description subjected to the action of body force in the isothermal condition for an incompressible Newtonian fluid, is represented by the continuity equation:

$$\nabla \cdot \vec{u} = 0 \tag{1}$$

and the momentum equation:

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nabla \cdot \left(\nu \nabla \vec{u}\right) + \vec{F} \tag{2}$$

where $\vec{u}$ is the velocity vector, $\vec{F}$ is the body force vector, p is the isotropic pressure, ρ is the density and ν is the kinematic viscosity. Pressure can be formally defined by the equation of state in the compressible flows, while for incompressible flows, it is derived from the divergence free condition of the velocity field. The classical projection method [26] is used to calculate the pressure field and enforce incompressibility, simultaneously. The discretized form of the momentum equation is split into two parts. The first being the prediction step and is based on viscous and body forces. In this step, the intermediate velocity field $\vec{u}^*$ is obtained from velocity at (n)th time step:

$$\frac{\vec{u}^* - \vec{u}^{(n)}}{\Delta t} = \left[\nabla \cdot \left(\nu \nabla \vec{u}\right) + \vec{F}\right]^{(n)}. \tag{3}$$

In each time step, the intermediate velocity field is calculated for fluid and boundary particles. In the second step, correction step, pressure force is included:

$$\frac{\vec{u}^{(n+1)} - \vec{u}^*}{\Delta t} = \left[-\frac{1}{\rho}\nabla p\right]^{(n+1)}. \tag{4}$$

The intermediate velocity field is usually not divergence free but this is imposed upon $\vec{u}^{(n+1)}$. Hence, the intermediate velocity is projected on the divergence free space by taking the divergence of Eq. (4) as:

$$\frac{\rho}{\Delta t}\nabla \cdot \left(\vec{u}^*\right) = \nabla^2 p^{(n+1)} \tag{5}$$

where the ∇^2 is the Laplacian operator. Once the pressure is obtained from Eq. (5), the velocity vector is updated by using the computed new pressure gradient:

$$\vec{u}^{(n+1)} = \vec{u}^* - \left(\frac{1}{\rho}\nabla p^{(n+1)}\right)\Delta t. \tag{6}$$

Finally, particles are moved according to this corrected velocity as:

$$r^{(n+1)} = r^{(n)} + \vec{u}^{(n+1)}\Delta t. \tag{7}$$

3. Method of discretization

The foundation of mesh free SPH method is based on integral interpolants which represents that any field variable X can be calculated over a set of SPH particles on domain of interest in terms of its values by taking a good interpolation kernel function. In discrete notation, the following approximation of the function at an interpolation particle a is lead:

$$X\left(\vec{r}\right) = \sum_b m_b \frac{X_b}{\rho_b} W_{ab} \tag{8}$$

where b is all the particles within the kernel function's support domain. m_b and ρ_b are the mass and density of particle b, respectively, and kernel function is denoted by $W_{ab} = W\left(\vec{r}_a - \vec{r}_b, h_0\right)$. The parameter h_0 is smoothing domain, and controls the size of the area around particle a where contribution from the rest of the particles cannot be neglected. Considering the computational accuracy and efficiency [27], here the cubic spline function in two dimensional is adopted.