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A multi-phase model of runaway core–mantle segregation in planetary embryos

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ABSTRACT

A classic scenario of core formation suggests that growing proto-planets are heated by the impacts of accreting planetesimals at their surface until their shallow layers reach the melting temperature of their metallic components or even of the silicates. In this partially molten shell, metal and silicates differentiate and the metallic phase ponds on top of the still undifferentiated inner planet. Later a gravitational instability brings dense metallic diapirs to the center of the planet. We test this multi-phase scenario by using a formalism that self-consistently accounts for the presence of solid silicates, solid and liquid iron. At each point of the mixture an average velocity and a separation velocity of the solid and liquid phases are defined. The energy balance accounts from the changes in potential energy associated with the segregation. We show that core formation starts before a significant melting of the silicates, as soon as impact heating is large enough to reach the melting temperature of the metallic component. Segregation proceeds in a few thousand years by a runaway process due to the conversion of gravitational energy into heat that occurs necessarily in all undifferentiated embryos of Moon to Mars sizes. The first metallic diapirs leave behind them a trailing conduit along which most of the further melting occurs. The cores of large planets do not form at the end of accretion but must result from the merging of the already differentiated hot cores of embryos.

1. Introduction

After condensation of the first solids in a nebula, the rocky grains coagulate near the central star to form small planetesimals (Kokubo and Ida, 1996). After a few 100 kyrs, the distribution of planetesimals is dominated by a few tens of moon-sized oligarchs (Kokubo and Ida, 1998). The terrestrial planets are then built by the violent merging of these oligarchs (Canup and Asphaug, 2001) resulting from their gravitational interactions. One of the late collision led to the formation of the Moon (around 60 Myr, Touboul et al., 2007). The segregation of Earth's core constrained by Hf–W timings has taken place with a mean age around 30 Myr (Yin et al., 2002) before the end of the accretion (around 100 Myr).

Small planetesimals have undergone early melting events due to the presence of short period radioactivities (Carlson and Langmuir, 2000). However, a widely accepted model initiates large scale core–mantle differentiation in planetary embryos from a shallow magma ocean generated by the heat deposited by the impacts of the accreting planetesimals (Kaula, 1979; Benz and Cameron, 1990; Tonks and Melosh, 1993; Rubie et al., 2003). In this partially molten shell, metal and silicates differentiate and the metallic phase ponds on top of the still undifferentiated inner planet. Later a gravitational instability brings dense metallic diapirs to the center of the planet (Stevenson, 1990). This scenario is supported by moderately siderophile element systematics (Li and Agee, 1996) and by simple physical arguments (Solomatov, 2000). However, up to now, it lacks a self consistent physical framework that requires simultaneously handling two components, metal and silicates, and two phases, solid and liquid (in this paper we reserve the word "phase" for the physical state, solid or liquid, and the word "component" for the chemical composition, metal or silicate). Previous attempts to simulate numerically the segregation, have made large simplifications, given the multi-phase non-Boussinesq mixture (i.e. where the density variations are comparable to the density itself) by considering it as a single Boussinesq fluid (e.g., Honda et al., 1993; Höink et al., 2006), or by considering that the whole metal component is liquid (e.g., Golabek et al., 2008). A number of issues remain therefore unclear, in particular, the dynamics of this segregation, continuous or punctuated, and whether the formation started in planetary embryos at the beginning of the oligarchic growth period or at the late stages of accretion. In this paper, we propose an attempt to answer these problems by using for the first time, a general multi-phase formalism that we developed in a series of papers (Bercovici et al., 2001; Bercovici and Ricard, 2003; Šrámek et al., 2007).

2. Impact energy and core segregation energy

It has been known for a long time that two energies are relevant to the post-impact dynamics. One is the energy buried by the impactor inside the planet (Tonks and Melosh, 1993), the other is the gravitational energy release by the core segregation (Flasar and Birch, 1973; Solomon, 1979).

When an impactor of mass m_i strikes a planet, a fraction f_1 of its kinetic energy is buried into a domain of mass $m = f_2 m_i$. The rest of the kinetic energy is rapidly radiated away and may heat up the primitive

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atmosphere (Matsui and Abe, 1986a). The two factors f_1 and f_2 are not well known but have been estimated from experiments and models to be f_1 ~ 1/3 and f_2 ~ 6, i.e., one third of the kinetic energy heats up rather homogeneously a volume 6 times larger than that of the impactor (Pierazzo et al., 1997; Monteux et al., 2007). The volume heated by the impact shock wave is roughly spherical and is tangent to the surface just below the impact point (Melosh, 1996). The escape velocity of a growing planet of radius R and surface gravity g, $\sqrt{2gR}$, should be indicative of the average impact velocity. We consider that the growing planet is undifferentiated with an average density $\bar{\rho}=\phi\,\rho_f+(1-\phi)\,\rho_m$ (ρ_f and ρ_m are the densities of metal and silicates, ϕ the volume proportion of metal). As $g=4/3\pi G\bar{\rho}R$, where G is the gravitational constant and the average density, the energy density transferred to the impacted planet (energy per unit mass of the heated zone) is therefore

$$\Delta e_1 = \frac{4\pi f_1}{3 f_2}\bar{\rho} G R^2. \quad (1)$$

This amounts to Δe_1 [J kg^{-1}]~6.4 10^{-2} R^2 [km^2]. For example, assuming the iron does not melt, this corresponds to a temperature increase of 260 K for a planet of 2000 km (all numerical values are listed in Table 1.). Alternatively, as soon as the planet reaches 1260 km, this energy δe_1 is enough to provide the latent heat $\rho_f \phi L/\bar{\rho}$ necessary to melt all the iron content of the impacted zone (L is the latent heat of iron melting and $\phi\,\rho_f/\bar{\rho}$ the mass proportion of metal).

The segregation of a undifferentiated planet with metal volume proportion ϕ and density $\bar{\rho}=\phi\,\rho_f+(1-\phi)\,\rho_m$ into a core of density ρ_f and a mantle of density ρ_m, is associated with a large change of gravitational energy (the gravitational energy is the generalization of the potential energy when the gravity field is time-dependent) and therefore releases the energy density (energy per unit mass of planet) (Flasar and Birch, 1973; Solomon, 1979)

$$\Delta e_2 = \frac{4G\pi R^2}{5\bar{\rho}}\left(\bar{\rho}^2 - \rho_f^2\phi^{5/3} - \rho_m^2\left(1-\phi^{5/3}\right) - \frac{5}{2}\left(\rho_f-\rho_m\right)\rho_m\phi\left(1-\phi^{2/3}\right)\right), \quad (2)$$

which cancels out for the three cases when segregation is not meaningful, $\phi=0$ (no metal), $\phi=1$ (no silicates) and $\rho_m=\rho_f$ (homogeneity). Typically for a planet containing 25% of metal in volume, Δe_2 [K]~5.8 10^{-2} R^2 [km^2] (see parameters in Table 1.). Before iron melts, this increases the temperature by ΔT_2~240 K for a planetary radius of $R=2000$ km.

Table 1
Typical parameter values for numerical models of two phase segregation.

Planet radius	R	2000 km
Silicate density	ρ_m	3200 kg m^{-3}
Iron density	ρ_f	7000 kg m^{-3}
Heat capacity	C	1 kJ K^{-1} kg^{-1}
Heat conductivity	k_T	3 W m^{-1} K^{-1}
Initial temperature	T_0	1100 K
Iron melting temp.	T_{melt}	1300 K
Initial metal content	ϕ_0	0.25
Silicate viscosity	μ_m	10^{19} Pa s
Solid iron viscosity	μ_m	10^{19} Pa s
Liquid iron viscosity	μ_f	1 Pa s
Permeability coeff.	k_0 $(k=k_0\phi^2)$	4 10^{-9} m^2
Average density $\bar{\rho}$	$\phi\,\rho_f+(1-\phi_m)\,\rho_m$	4150 kg m^{-3}
Gravity	$g=4\pi G\bar{\rho}R/3$	2.32 m s^{-2}
Temperature excess	ΔT	258 K
Temperature scale	$\theta=\Delta\rho g R/\rho C$	4247 K
Stokes velocity scale	$\Delta\rho g R^2/\mu_m$	111 km/yr
Time scale	$\mu_m/\Delta\rho g R$	18 yr
Darcy velocity	$k_0\Delta\rho g\phi_0^2\,(1-\phi_0)/\mu_f$	58 m/yr
Compaction length	$\sqrt{k_0\mu_m/\mu_f}$	210 km
Norm. comp. length	δ	0.1
Rayleigh number Ra	$\bar{\rho}\Delta\rho g C_p R^3/\eta_m k_T$	10^{10}

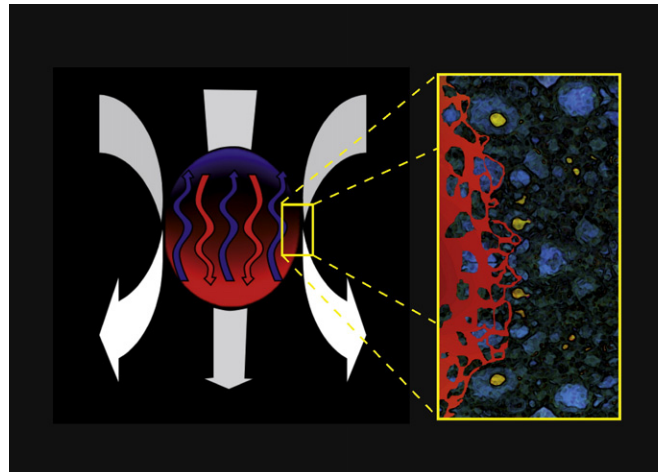

Fig. 1. Principle of the multi-phase formalism: at microscopic scale (shown in the insert), the undifferentiated planet is either cold and made of solid silicates and solid metal (right side), or above the metal melting temperature, T_{melt}, and made of solid silicates (blue) and liquid metal (red). This microscopic physics when averaged over continuous variables leads to a macroscopic flow, superposition of a usual Stokes flow (white arrows) and a relative flow that segregates the dense metal (red) from the light residual silicates (blue), within the molten area. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The R^2 dependences and the comparability of the impact and core segregation energy densities have two consequences. Firstly, melting upon impact is rapidly inescapable as the energy density Δe_1 brought by impacts increases rapidly with R. Secondly, the differentiation of a given planetary volume initiated by an input of energy Δe_1 is potentially able to release enough gravitational energy, Δe_2, to melt the metal component of an equivalent undifferentiated volume. The first impact that induces metal melting can therefore initiate a runaway differentiation of a planet.

3. Multiphase equations for a metal-silicate continuum

A classical averaging approach combined with symmetry arguments is used to derive the mass, momentum, and energy equations of a mixture made of three different interacting materials: solid silicates, solid metal and liquid metal (McKenzie, 1984; Bercovici et al., 2001; Ricard et al., 2001; Bercovici and Ricard, 2003; Šrámek et al., 2007) (see Fig. 1). The three individual materials are considered as viscous, but the viscosities of the so called solid materials (the silicate and metal components in the solid phase) are infinitely larger (and equal for simplicity) than that of the liquid metal. At each point of the multiphase continuum the metal component is either totally in the solid state or totally liquid, but the metal can be solid in some parts of the planet and liquid in other parts. The simultaneous presence of two very different viscosities is a major numerical difficulty. We have not considered more complex rheologies like viscoelasticity or viscoplasticity.

The two components, metal and silicate, in volume proportions ϕ and $1-\phi$ have properties denoted by the subscripts f and m. These subscripts are in agreement with the previous derivation of two phase equations (Bercovici and Ricard, 2003; Šrámek et al., 2007) although in the present paper, the metallic phase (subscript f) is not necessarily fluid. The metal density, solid or liquid, is ρ_f and we call $\Delta\rho$ and Δv the differences of density and of volume averaged velocities between the silicate and metal components, $\rho_m-\rho_f$ and v_m-v_f. At each point in space, we define both a silicate velocity v_m and a metal velocity v_f that can be equal (when the metal is solid and locked in the silicates) or different (when the fluid metal can separate from the silicates). We non-dimensionalize lengths by the planet radius R, velocities by the two-phase Stokes velocity $|\Delta\rho|\,gR^2/\mu_m$, time by $\tau=\mu_m/(|\Delta\rho|gR)$, pressures by $|\Delta\rho|gR$ and temperatures by $\theta=|\Delta\rho|gR/(\bar{\rho}\bar{C})$.