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Normal contact with high order finite elements and a fictitious contact material

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ABSTRACT

Contact problems in solid mechanics are traditionally solved using the *h*-version of the finite element method. The constraints are enforced along the surfaces of e.g. elastic bodies under consideration. Standard constraint algorithms include penalty methods, Lagrange multiplier methods and combinations thereof. For complex scenarios, a major part of the solution time is taken up by operations to identify points that come into contact. This paper presents a novel approach to model frictionless contact using high order finite elements. Here, we employ an especially designed material model that is inserted into two- respectively three-dimensional regions surrounding contacting bodies. Contact constraints are thus enforced on the same manifold as the accompanying structural problem. The application of the current material formulation leads to a regularization of the Karush–Kuhn–Tucker conditions. Our formulation can be classified as a barrier-type method. Results are obtained for two- and three-dimensional problems, including a Hertzian contact problem. Comparisons to a commercial FEA package are provided. The proposed formulation works well for non-matching discretizations on adjacent contact interfaces and handles self-contact naturally. Since the non-penetrating conditions are solved in a physically consistent manner, there is no need for an explicit contact search.

1. Introduction

The simulation of mechanical contact phenomena is of interest in various fields of engineering. Mechanical systems usually consist of several parts, and their interaction needs to be represented correctly.

Numerical solutions for contact problems are mostly obtained by applying the finite element method (FEM) [1,2]. The overwhelming part of the literature is concerned with the classical *h*-version of the FEM, which uses linear or at most quadratic ansatz functions for spatial discretization. Contact constraints are enforced along contact interfaces defined on the surfaces of e.g. elastic bodies under consideration. Early discretizations of these interfaces were based on so-called *node-to-node* contact elements, for which the nodal positions of adjacent bodies had to match at the interface [3–5]. Because of the limited applicability of this approach, e.g. *node-to-segment* contact elements were developed allowing the nodes to slide along the whole contact interface [6,7]. However, non-smooth surfaces, which typically arise in linear FE meshes for the *h*-version, pose a challenge for this kind of formulation. For problems involving large sliding, nodes can get stuck at re-entrant corners. To overcome this issue, mortar-type methods have been developed, which enforce contact constraints in a weak (integral) sense [8–15].

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Recently, high order discretizations have been introduced to the realm of contact mechanics. These discretizations offer advantages concerning the overall accuracy of the numerical approximation as well as a smoother and, thus, improved description of the contact interface. The contributions made cover the p-version of the finite element method [16–18] and NURBS basis functions used in isogeometric analysis (IGA) [19,20].

Different possibilities have been developed for the enforcement of contact constraints. The predominant approaches are the penalty method, the Lagrange multiplier method and combinations thereof [21,22]. Furthermore, barrier methods [23], Nitsche's method [24,25] and the formulation of Dirichlet–Neumann problems [26] have been applied successfully.

Since contact constraints are generally of inequality type, even problems of linear elasticity are rendered non-linear. Hence, these problems are generally solved iteratively [21]. Here, each iteration is characterized by a global search for bodies likely to come into contact, a local search at element level for changes in the state of contact such as penetration or stick/slip, and the actual enforcement of contact constraints. For complex contact scenarios, the search for contact can make up a major part of a simulation [1].

In recent years, formulations have been introduced, which utilize alternative discretizations for the contact interface or employ new schemes to enforce contact constraints. Oñate et al. proposed an approach for contact modeling in the context of the *particle element method* [27]. Here, nodes are regarded as particles carrying all relevant solution information. In every time step, a new mesh is created from the positions of the particles using a Delaunay triangulation to solve the elasticity problem by means of linear finite elements. When the mesh is created, contact elements are identified as simplices that connect different bodies or are attached to a rigid surface. Inside these contact elements, internal forces are applied to avoid normal penetration and to model friction.

Another approach, introduced by Oliver and co-workers, is the *contact domain method* [28,29]. Their idea is to introduce a contact mesh of linear triangles – the contact domain – between bodies in close proximity. This contact mesh is then used to establish normal and tangential gap measures. Contact constraints are enforced using Lagrange multipliers.

A formulation similar to the one followed in the publication at hand, yet using the h-version of the FEM, was introduced as the *third medium approach* by Wriggers et al. [30]. Here, the authors embed elastic bodies in a special contact material. This material consists of a highly compliant isotropic part and an anisotropic part. The latter is activated once the contact material is compressed below a certain threshold.

In this paper, we propose an approach that combines a material formulation to enforce contact constraints and a high order discretization, namely the p-version of the finite element method. Our contribution is motivated by the *finite cell method* (FCM), an embedded domain method based on high order finite elements [31,32]. Alike FCM, this contact formulation embeds the original domain of computation in a larger 'fictitious' domain, yet here not considered as 'void' material but as a non-linear medium to avoid penetration of surfaces. The application of a contact material enforces contact constraints in a physically consistent manner by relying on the growth condition [33] and can, thus, be classified as a barrier type method [21]. Also, there is no need for an explicit contact search, since the contact status is based on the deformation of the contact material. In contrast to the work by Wriggers et al. [30], our formulation only demands for a single material parameter and employs a consistent linearization. Furthermore, by using the p-version of the FEM, complex deformation scenarios can be recovered using only few coarse elements. As this approach can readily be combined with the finite cell method, it simplifies mesh generation significantly, as e.g. voids or fillets in the structure need not be explicitly represented by element boundaries.

The present work is outlined as follows: Section 2 recalls the basic features of the p-version of the finite element method and the finite cell method. Section 3 introduces a formulation for the enforcement of contact constraints using a contact material. The new approach will be illustrated by numerical examples in Section 4. The paper closes with some concluding remarks in Section 5.

2. High order finite elements and the finite cell method

The current section gives a short overview of the p-version of the finite element method as well as the finite cell method (FCM). The latter is fundamental to the contact approach outlined in this publication and is used in some of the examples presented in Section 4 for modeling small geometric features such as voids or fillets. Here, the overview is limited to the extent necessary to understand the rest of the paper. For further details on p-FEM and FCM the reader is referred to [34,35,31,32].

2.1. High order finite elements (p-FEM)

The calculations in this publication have been performed using the p-version of the finite element method. The main benefits of the p-version are exponential convergence for smooth problems, insensitivity to mesh distortion and well-behaved condition numbers for increasing polynomial orders. Just like conventional finite elements, the p-version of the FEM is based on a weak form of the partial differential equation under consideration. Fitting the scope of the paper at hand, this shall be exemplified for the case of hyperelasticity by applying the principle of virtual work

$$\delta \mathcal{W}(\mathbf{u}, \delta\mathbf{u}) = \int_{\Omega} \boldsymbol{\sigma} : \delta\boldsymbol{\epsilon}\, dv - \int_{\Omega} \mathbf{f} \cdot \delta\mathbf{u}\, dv - \int_{\Gamma_N} \mathbf{t} \cdot \delta\mathbf{u}\, ds = 0. \tag{1}$$