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Weak Dirichlet boundary conditions for trimmed thin isogeometric shells

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ABSTRACT

Computer-aided design-based NURBS surfaces form the basis of isogeometric shell analysis which exploits the smoothness and higher continuity properties of NURBS to derive a suitable analysis model in an isoparametric sense. Equipped with higher order approximation capabilities the used NURBS functions focus increasingly on rotation-free shell elements which are considered to be difficult in the traditional finite element framework. The rotation-free formulation of shell elements is elegant and efficient but demands special care to enforce reliably essential translational and rotational boundary conditions which is even more challenging in the case of trimmed boundaries as common in CAD models. We propose a Nitsche-based extension of the Kirchhoff–Love theory to enforce weakly essential boundary conditions of the shell. We apply our method to trimmed and untrimmed NURBS structures and illustrate a good performance of the method with benchmark test models and a shell model from engineering practice. With an extension of the formulation to a weak enforcement of coupling constraints we are able to handle CAD-derived trimmed multi-patch NURBS models for thin shell structures.

1. Introduction

The conceptual ideas behind isogeometric analysis (IGA) aim at unifying computer aided design (CAD) and finite element analysis (FEA) [1,2]. Isogeometric analysis employs the *non-uniform rational B-spline functions* (NURBS) [3] used for the geometric description of the structure to approximate its physical response in an isoparametric sense. In comparison with the finite element method isogeometric analysis behaves superior in many fields which can be mainly attributed to the exact geometry representation, the combination of higher continuity and higher order approximation properties and the method's unique refinement capabilities. The superiority of the method has been demonstrated by a number of researchers and meanwhile covers all fields of numerical analysis and simulation, see e.g. [4–17].

Shell structures excel by an optimal load-carrying behavior and are of major importance in the design of structures in aerospace and automotive engineering. The membrane-like geometry of shells is described perfectly by NURBS which makes them attractive for the isogeometric paradigm where the geometric and parametric properties of NURBS surfaces are fully exploited. The geometrically reduced description of thin shell structures by the shell mid-surface is harnessed by various established mathematical models used to describe their deformation behavior [18–20]. Kiendl et al. [21] proposed a rotation-free isogeometric shell element based on the theory of Kirchhoff–Love which was later extended by Nagy et al. [22] for the design of anisotropic composite shell structures. Other important developments include the isogeometric Reissner–Mindlin

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element proposed in [23–25], the hierarchic family of shell elements proposed in [15] to overcome classical locking phenomena and solid-like shell formulations as proposed in [14,26,27].

The absence of rotational degrees of freedom in rotation-free shells request for a special treatment of essential boundary conditions. A reliable strong enforcement of these kinematic boundary conditions is only possible in a limited number of special cases where a sufficient number of the, in general, non-interpolatory control points exactly influence the considered boundary part and where trimmed boundaries remain unconsidered. The usual approach includes constraint degrees of freedom at the control points along the boundary and, in addition, along a boundary neighboring row of control points inside the shell domain [25,28]. In [8] it is shown that clamped or symmetry boundary conditions can be maintained if the direction of the tangent normal to the geometry boundary is preserved during deformation. In cases where the boundary is part of a trimmed geometry this approach may not be sufficient or may even fail completely. The trimming concept used in CAD software is a common concept to simplify the model definition for visualization purposes. Defined by a trimming curve on the governing domain patch the approach specifies regions to be faded out in the graphical representation [29,30]. This way increasing visual complexity is introduced while the geometric complexity is restricted to the shape of the underlying NURBS surface. In [26,31] the trimming concept was assigned to the isogeometric analysis model. The authors replaced trimmed areas of the graphical representation by a fictitious domain in the analysis domain. The fictitious domain concept was adapted from the finite cell method (FCM) which was originally introduced on Cartesian grids on the basis of the hierarchical p-FEM approximation space [32–41].

As a consequence of the restrictive modeling options for essential boundary conditions we consider an approach which weakly enforces boundary constraints. Besides the popular penalty method [42,43] and the Lagrange multiplier method [44–47], methods based on the idea of Nitsche [48–50] have gained much attention in the framework of isogeometric analysis [31,37,51–56].

In this contribution we focus on Nitsche's method to enforce weakly essential boundary conditions of rotation-free Kirchhoff–Love shell structures including boundaries of trimmed models. We extend the Kirchhoff–Love shell formulation variationally consistent, including transverse shear components of the stress tensor, and provide the complete set of equations governing the elasticity problem. We choose adequate stabilization terms which are tailored to the need of the shell problem and which ensure optimal convergence properties for the analysis. Finally, we show that the conceptual approach can be extended easily to a formulation which enforces coupling constraints for isogeometric trimmed multi-patch models in a corresponding manner [57,58]. We provide a number of examples to demonstrate the overall performance of our method and its enhanced flexibility based on the finite cell method.

The paper is organized as follows: we start with a brief summary of non-uniform rational B-splines in Section 2 followed by a concise presentation of the Kirchhoff–Love shell theory in Section 3. The Nitsche extension used in our formulation is introduced in Section 4.1. A short summary of the finite cell method and the specific boundary treatment in case of NURBS trimming curves is provided in Section 5. Several examples are presented in Section 6. Finally, we summarize the main findings and draw conclusions in Section 7.

2. Non-uniform rational B-splines

Isogeometric analysis applies the non-uniform rational B-splines (NURBS) used to represent the geometry of the analysis structure to approximate the physical field and state variables in an isoparametric sense [1]. NURBS are a generalization of B-splines and follow from a projective transformation of B-spline entities in $\mathbb{R}^d$, $d = \{1, 2, 3\}$ [3,59] which introduces weights w_m $(m = 1, \ldots, M)$ as form parameters that control the NURBS shape. The definition of the shell mid-surface with NURBS follows from a linear combination of control points $\mathbf{P}_m \in \mathbb{R}^3$ with the respective NURBS basis functions

$$\mathbf{x}(\xi, \eta) = \sum_m R_{m,p}(\xi, \eta)\, \mathbf{P}_m \tag{1}$$

where $m = m(i, j)$ and where

$$R_{m,p}(\xi, \eta) = \frac{w_m\, N_{i,p_1}(\xi)\, M_{j,p_2}(\eta)}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{n} w_{\hat{m}}\, N_{\hat{i},p_1}(\xi)\, M_{\hat{j},p_2}(\eta)} \tag{2}$$

is a multi-variate NURBS basis function of degree p. The functions $N_{i,p_1}(\xi)$ $(i = 1, \ldots, n)$, are one-dimensional B-splines of polynomial degree p_1 defined in the parameter space Ξ which is specified by a knot vector

$$\Xi = \{\xi_1, \ldots, \xi_{n+p+1}\}, \quad \xi_1 \le \xi_2 \le \cdots \le \xi_{n+p+1} \tag{3}$$

consisting of a non-decreasing sequence of coordinates ξ_i, denoted as knots. The functions $M_{j,p_2}(\eta)$ follow in analogy to $N_{i,p_1}(\xi)$. B-spline functions are defined piecewise over $p + 1$ knot-spans. They form a $\mathcal{C}^{p-1}$ continuously differentiable basis and can be constructed by the *Cox–de-Boor* recursion formula [3,28]. Repeated knots lower the continuity of the basis functions. A knot multiplicity of $p + 1$ for the first and last knot makes the basis interpolatory resulting in a B-spline patch with *open* knot vector, cf. Fig. 1.