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A monolithic finite element approach using high-order schemes in time and space applied to finite strain thermo-viscoelasticity

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ABSTRACT

This article addresses a thermo-mechanically coupled problem of thermo-viscoelasticity at finite strains using a monolithic approach. The underlying equations are based on the non-linear transient heat equation, the local equilibrium conditions and the evolution equations of the internal variables. The latter describe the hardening behavior of the material. If the method of vertical lines is applied, its first step – namely the spatial discretization – yields a system of differential-algebraic equations (DAE-system). Here, we employ the p-version of the finite element method based on integrated Legendre polynomials. This can lead to very precise solutions in the spatial domain. In order to be accurate in the time-domain as well, stiffly accurate, diagonally-implicit Runge–Kutta methods are applied to solve the DAE-system yielding a coupled system of non-linear algebraic equations. In this article, the system is solved monolithically by employing the Multilevel-Newton algorithm. Accordingly, a high-order result is obtained in the space and the time domain. The numerical concept is applied to a constitutive model of finite strain thermo-viscoelasticity. Several examples are applied to demonstrate the efficiency and applicability of the numerical scheme. It is especially the transient problems that call for time-adaptive schemes which are naturally embedded in the concept.

1. Introduction

Thermo-mechanically coupled computations are of great importance if either the heat equation is influenced by the deformation or if the material properties are temperature sensitive. The first case occurs either when internal mechanical dissipation generates heat – such as in metal forming or in cyclic processes, respectively – or in large deformation cases where the entire surface changes so significantly that the amount of heat of the material body is essentially influenced. In the second case, the material properties depend on temperatures during hot forming processes, see, for instance, [1–3] and the literature cited therein, or are connected to polymers and rubber-like material that are highly temperature-sensitive. In the field of rubber-elasticity, first numerical attempts were made by Holzapfel [4], Holzapfel and Simo [5], Lion [6], Lion [7], and Reese [8]. Holzapfel [4] is an introductory textbook dealing with entropic elastic material behavior. While Lion [6,7] addresses finite thermo-viscoplastic material behavior, the derivation of a thermodynamically consistent material model for finite thermo-viscoelastic material behavior is discussed in [8]. For additional information on this topic see also [9]. In [8], the spatial discretization is carried out with elements based on a stabilized reduced integration technique that enables

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large deformations. For reduced integration techniques in the field of finite thermo-viscoelastic material behavior, see [10]. The employed variational formulation contains a parameter which is, to a certain extent, artificial. Lion [11] addresses the investigation and modeling of a filler loaded tread compound. The material behavior is characterized by non-linear elasticity in combination with non-linear rate-dependence. Furthermore, a weak equilibrium hysteresis can be observed. Accordingly, it exhibits the characteristics of a thermo-viscoplastic material. In [12], a thermo-viscoelastic material model for large strain applications is discussed, drawing on a monolithic approach employing a Backward-Euler time integration scheme. Miehe [13] considers problems of large strain rate-independent multi-surface thermo-plasticity. The governing equations are solved by means of an isothermal operator split—implying that, firstly, the isothermal sub-problem at frozen temperature is solved, and that, secondly, the heat conduction problem at frozen configuration is computed. The spatial discretization is carried out by means of Q1P0-elements, see [14]. The topic of large strain thermo-plasticity is also addressed in [15]. Miehe [16] deals with entropic thermoelastic material behavior at finite strains. On the one hand an isothermal and on the other hand an isentropic deformation predictor is taken into account. In both cases a purely thermal heat conduction corrector phase is introduced after the predictor phase in order to solve the governing equations. Heimes [17] treated the case of finite strain thermo-viscoelasticity experimentally, adapting a constitutive modeling concept and implementing it into a commercial finite element program. All investigations are related to a "linear" discretization scheme, both in space and in time. Now, the question is how to develop a program using high-order space and time-discretization.

Since structures are influenced by mechanical and thermal loads, we are interested in the numerical solution of the aforementioned problems. In finite element computations the numerical treatment can be divided into partitioned and monolithic approaches. In a partitioned approach both fields are solved by different programs. The advantage is that each program can be handled separately, even with different discretization schemes. The coupling has to be treated very carefully – both for the time- and the space-discretization as well as for the coupling itself – because no heat should be generated or lost due to the numerical algorithm, as this can lead to unstable computations. Monolithic approaches have the advantage to be completely independent of such problems and no coupling program has to be developed. One common disadvantage is that the same discretization is used for each field (temperature and displacements), and that the resulting non-linear system is larger. However, it is – even in one program – not necessary to have the same discretization schemes, meaning the same mesh and/or the same polynomial degree. If different spatial integration points occur, the mapping of the internal variables must be discussed in more detail, see [18].

It is very common to apply numerical discretization methods of low order, both for the space and for the time-discretization. To overcome locking phenomena in linear finite element approximations, various mixed element formulations or stabilization techniques have been developed, but they do not really solve the problem that the convergence to the exact solution is not sufficient for decreasing the element-size. The element increase using linear (or quadratic elements) is called "h-version of finite elements", whereas "h-extension" defines the mesh refinement. Caused by these limited convergence properties, see [19,20], we make use of hierarchical shape function based on integrated Legendre-polynomials. In this respect, we keep the mesh fixed and increase the polynomial degree. This is called the "p-extension". Naturally, decreasing the element-size and increasing the polynomial order can be called "hp-extension". In the context of high-order elements, two main branches have been followed. The first is based on the use of integrated Legendre-polynomials, see [21–25]. This approach is associated with the p-version of the finite element method which is generalized to finite strains in [19,26–28]. In the second branch, iso-geometric element formulations are proposed, see e.g. [29]. In CAD-systems, which provide the polynomial description of the shape function, however, it is common to apply polynomial degrees of third order. Even here, an hp-extension can be introduced, which, however, violates the original idea of an isogeometric formulation.

Within the last three decades the p-extension has been shown to be a very competitive approach to classical h-extension finite elements. Low order elements have the disadvantage of their convergence behavior, and of showing volumetric locking phenomena for nearly incompressible materials. In the case of the p-extension, it is shown that above a certain polynomial degree, locking phenomena decrease, see [26]. In [21] it is demonstrated numerically that the error measured by means of the energy norm approaches zero at a certain rate, independent of the Poisson ratio. This was considered for problems of small strain elasticity. Theoretical considerations are discussed in [30]. For further readings concerning the p-extension in the context of small strain elasticity see e.g. [31,32]. Problems of large strain (isotropic) hyper-elasticity in conjunction with the p-extension are addressed in [19,33] or [20]. The superior characteristics of the p-extension in the context of the locking phenomenon also hold true in the large strain regime, see [27,26]. For an extension to problems of anisotropic hyper-elasticity, see [34].

The topic of small strain plasticity is treated in [35], whereas problems of large strain powder plasticity are discussed in [36,28]. For the application of the p-extension for the spatial discretization to problems of finite strain viscoelasticity see [20], where diagonally-implicit Runge–Kutta schemes (DIRK-schemes) as well as Rosenbrock-type schemes are used, both employing an embedded time step-size control for the temporal discretization. An extension of the p-version FEM to coupled problems of finite strain thermo-elasticity, or electro-thermoelasticity is given in [37,38], where order one methods in the time domain are applied. For stationary, one-dimensional considerations, see [39] as well.

Here, we follow a decisive concept of a discretization in space and time. According to Wittekindt [40] and Fritzen [41] the classical finite element approach can be related to the method of vertical lines. First of all, the spatial discretization is carried out by means of finite elements yielding a system of differential algebraic equations (DAE-system). The algebraic part of the DAE-systems consists of the spatially discretized weak form of the equilibrium condition, which constitutes a system of non-linear equations in the coefficients contained in the ansatz for the displacement and temperature field, here in the sense