Computers and Mathematics with Applications 70 (2015) 1–10

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Numerical option pricing without oscillations using flux limiters

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ARTICLE INFO

Article history:
Received 15 August 2014
Received in revised form 12 February 2015
Accepted 4 April 2015
Available online 12 May 2015

Keywords:
Black–Scholes equation
Option pricing
Discontinuities
Flux limiters

ABSTRACT

A numerical method is developed for the solution of the Black–Scholes equation avoiding the oscillations that are common close to a discontinuity in the pay-off function. Part of the derivatives are evaluated explicitly and part of them are computed implicitly using operator splitting. The method is second order accurate in time and almost of second order in the asset price for smooth solutions and no system of nonlinear equations has to be solved. A flux limiter modifies the first derivative in the equation such that no oscillations occur in the solution in the numerical examples presented.

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1. Introduction

In this note we consider the numerical solution of the Black–Scholes equation [1,2]

$$u_t(t,s) + \mathcal{L}u = u_t(t,s) + \mathcal{F}u + \mathcal{G}u = 0, \quad s \in \mathbb{R}^+,\ 0 \le t \le T, \tag{1}$$

where

$$\begin{cases} \mathcal{F}u &= -rsu_s(t,s), \\ \mathcal{G}u &= -\frac{1}{2}\sigma^2(t,s)s^2u_{ss}(t,s) + ru(t,s), \end{cases} \tag{2}$$

and a subscript s or t denotes derivation with respect to the variable. The solution u to (1) and (2) determines the price of an option issued on the underlying asset s. The parameter $r \in \mathbb{R}^+$ is the short rate of interest and $\sigma \in \mathbb{R}^+$ determines the volatility of s. We have transformed the original final-value problem to an initial-value problem in forward time for simplicity of notation. The initial condition is given by

$$u(0,s) = \Phi(s), \tag{3}$$

where $\Phi(s)$ is the so called pay-off function, i.e. the value of the option at time of maturity.

For many classes of problems, Φ and/or its derivatives are discontinuous. Standard time-integration schemes such as the Crank–Nicolson method [3] often fail to produce accurate numerical solutions due to oscillations arising from the discontinuities. For this reason the so called Rannacher scheme [4] is often employed [5,6]. In this method a number of initial time-steps with the backward Euler method are preceding the Crank–Nicolson method in order to damp the oscillations

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http://dx.doi.org/10.1016/j.camwa.2015.04.003
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occurring from the discontinuities in the pay-off function. However, for interest rate dominated problems, i.e. when r is significantly larger than σ, also the Rannacher scheme gives rise to oscillations in the solution.

In the 1970s so called flux limiters were introduced to handle shocks present in fluid flow problems, see e.g. [7], [8, p. 115]. This methodology has also been utilized for option pricing problems [9,10]. Problems that are of convection–diffusion type like (1) often needs to be discretized implicitly in time to avoid a severe restriction on the time step due to the second derivative in (1). The flux limiters that are effective for problems with discontinuities are nonlinear, including maxima, minima, and absolute values. A fully implicit discretization introduces an iteration in the scheme because of the nonlinearity. This complicates the solution procedure considerably since Newton iterations are not applicable.

To obtain a method that can handle discontinuities, yields stable discretizations in time, and does not introduce a nonlinear iteration in each time step, we will use operator splitting in time [11]. It is of second order accuracy in time and almost of second order in the asset price and is parameter-free. The part $\mathcal{G}$ that includes second derivatives is treated implicitly in time for stability reasons while the first order part $\mathcal{F}$ is treated explicitly. This way, flux limiters can be used for $\mathcal{F}$ where it is needed to avoid oscillatory solutions, still not causing nonlinearities on the implicit side. The scheme can be extended to more dimensions in the asset price by dimensional splitting of $\mathcal{F}$ as is common in computational fluid dynamics [8, ch. 19.5]. It can also be applied to other types of pay-off functions of low regularity than those tested here.

The outline of the rest of the paper is as follows. In Section 2, the discretization in space and time is presented. The method is analyzed with respect to accuracy and stability in Section 3 and numerical results for a European call option and a binary option are presented in Section 4. Finally we discuss our method and draw conclusions in Section 5.

2. Discretization

We consider a discretization of (1) in the asset price and time on a structured and equidistant grid (t^n, s_j), $t^n = n\Delta t$, $s_j = jh$. The operator $\mathcal{G}$ is approximated using centered second order finite differences

$$\mathcal{G}u_j = -\frac{1}{2}\sigma_j^2 s_j^2 \frac{u_{j+1} - 2u_j + u_{j-1}}{h^2} + ru_j. \tag{4}$$

To avoid spurious oscillations we use flux limiters for the first order operator $\mathcal{F}$ as in [8, Ch. 6]

$$\mathcal{F}u_j = -rs_j\chi\frac{\Delta_+ u_j - (f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}})}{h}, \tag{5}$$

with

$$\begin{aligned} &\Delta_+ u_j = u_{j+1} - u_j, \qquad \chi = 1 + \frac{1}{2}\Delta t r, \\ &f_{j-\frac{1}{2}} = \frac{1}{2}\left(1 - \frac{\Delta t}{h} r s_j \chi\right)\delta_{j-\frac{1}{2}}, \qquad f_{j+\frac{1}{2}} = \frac{1}{2}\left(1 - \frac{\Delta t}{h} r s_j \chi\right)\delta_{j+\frac{1}{2}}. \end{aligned} \tag{6}$$

Here

$$\delta_{j+\frac{1}{2}} = \phi(\Delta_+ u_{j+1}, \Delta_+ u_j), \tag{7}$$

and ϕ is the so called flux limiter function. We consider the minmod-limiter defined by

$$\phi(a,b) = \begin{cases} 0, & ab \le 0, \\ a, & \text{if } |a| < |b|, \\ b, & \text{if } |b| < |a|. \end{cases} \tag{8}$$

In time we will employ a Strang operator splitting [11] that allows us to treat $\mathcal{G}$ implicitly in time and $\mathcal{F}$ explicitly. The implicit scheme to solve $u_t + \mathcal{G}u = 0$ from t^{n-1} to t^n is the TR-BDF2 [12,13]

$$\begin{aligned} 1.\quad \tilde{u}_j^n &= u_j^{n-1} - \frac{\Delta t}{4}\left(\mathcal{G}u_j^{n-1} + \mathcal{G}\tilde{u}_j^n\right), \\ 2.\quad u_j^n &= \frac{1}{3}\left(4\tilde{u}_j^n - u_j^{n-1} - \Delta t\mathcal{G}u_j^n\right). \end{aligned} \tag{9}$$

The explicit scheme to solve $u_t + \mathcal{F}u = 0$ from t^{n-1} to t^n is

$$u_j^n = u_j^{n-1} - \Delta t\mathcal{F}u_j^{n-1}. \tag{10}$$

A nonlinearity is introduced in $\mathcal{F}$ by the flux limiter (8) but it is applied explicitly in (10) to u_j^{n-1} with low additional computational complexity. The limiter will be active to avoid oscillatory behavior close to near discontinuities in the solution.

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