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Global regularity of the 2D magnetic micropolar fluid flows with mixed partial viscosity☆

Jianfeng Cheng [a], Yujun Liu [a,b,*]

[a] Department of Mathematics, Sichuan University, Chengdu 610064, PR China

[b] Department of Mathematics and Computer, Panzhihua University, Panzhihua 617000, PR China

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ABSTRACT

In this paper, we study the Cauchy problem for the 2D anisotropic magneto-micropolar fluid flows with mixed partial viscosity. We established the global regularity of the 2D anisotropic magneto-micropolar fluid flows with vertical kinematic viscosity, horizontal magnetic diffusion and horizontal vortex viscosity.

1. Introduction and main results

The 3D magneto-micropolar fluid flows can be stated as follows

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} + \nabla\pi = (\mu+\chi)\triangle\mathbf{u} + \mathbf{b}\cdot\nabla\mathbf{b} + 2\chi\nabla\times\mathbf{m}, \\ \partial_t \mathbf{b} + \mathbf{u}\cdot\nabla\mathbf{b} = \nu\triangle\mathbf{b} + \mathbf{b}\cdot\nabla\mathbf{u}, \\ \partial_t \mathbf{m} + \mathbf{u}\cdot\nabla\mathbf{m} + 4\chi\mathbf{m} = \kappa\Delta\mathbf{m} + (\alpha+\beta)\nabla\nabla\cdot\mathbf{m} + 2\chi\nabla\times\mathbf{u}, \\ \nabla\cdot\mathbf{u} = \nabla\cdot\mathbf{b} = 0, \end{cases} \tag{1.1}$$

where $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{b} = (b_1, b_2, b_3)$, $\mathbf{m} = (m_1, m_2, m_3)$ and π denote the velocity field, the magnetic field, the micro-rotation field and the scalar pressure, respectively. The nonnegative constants μ and ν are the Newtonian kinetic viscosity and the magnetic diffusion coefficient respectively. The parameter $\chi > 0$ is the dynamic micro-rotation viscosity, the non-negative constants $\alpha,\ \beta$ and κ are the angular viscosities.

The magnetic micropolar fluid flows describe a micropolar fluid which is moving into a magnetic field (see [1]). Ahmadi and Shahinpoor investigated the stability of the solution to the 3D magnetic micropolar fluid in [1]. Fortunately, the local existence and uniqueness of strong solution were proved by Rojas-Medar in [2]. However, the global regularity of the strong solution with large initial data or finite time singularity to the 3D magneto-micropolar fluid equations is still an open problem. Recently, many mathematicians investigated the blow-up criteria of the strong solutions to the 3D magneto-micropolar fluid equations in [3–6]. As an intermediate case, we want to study the 2D magnetic micropolar fluid equations

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* Corresponding author at: Department of Mathematics, Sichuan University, Chengdu 610064, PR China.
E-mail addresses: jianfengcheng@126.com (J. Cheng), liuyujun6@sohu.com (Y. Liu).

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with partial viscosity. For the 2D case, the velocity field and magnetic field can be understood as $\mathbf{u} = (u_1(x, y, t), u_2(x, y, t), 0)$ and $\mathbf{b} = (b_1(x, y, t), b_2(x, y, t), 0)$. We assume that the rotational axis of particles is parallel to the Z-axis, namely, $\mathbf{m} = (0, 0, m(x, y, t))$. Inserting $\mathbf{u}$, $\mathbf{b}$ and $\mathbf{m}$ of above forms into Eqs. (1.1), one has

$$\begin{cases} \partial_t u + u\cdot\nabla u + \nabla\pi = (\mu+\chi)\Delta u + b\cdot\nabla b + 2\chi\nabla^{\perp} m, \\ \partial_t b + u\cdot\nabla b = \nu\Delta b + b\cdot\nabla u, \\ m_t + u\cdot\nabla m + 4\chi m = \kappa\Delta m + 2\chi\nabla\times u, \\ \nabla\cdot u = \nabla\cdot b = 0, \end{cases} \tag{1.2}$$

where $u = (u_1(x, y, t), u_2(x, y, t))$, $b = (b_1(x, y, t), b_2(x, y, t))$ and $\nabla^{\perp} m = (m_y, -m_x)$.

The system (1.2) is the 2D micropolar fluid equations when the magnetic b is taken to zero, which is expressed as

$$\begin{cases} \partial_t u + u\cdot\nabla u + \nabla\pi = (\mu+\chi)\Delta u + 2\chi\nabla^{\perp} m, \\ \partial_t m + u\cdot\nabla m + 4\chi m = \kappa\Delta m + 2\chi\nabla\times u, \\ \nabla\cdot u = 0. \end{cases} \tag{1.3}$$

For the global regularity of the 2D micropolar fluid equations, Lukaszewicz in [7] obtained the global well-posedness of classical solutions for 2D micropolar fluid with full viscosity. However, whether or not the classical solution of the 2D inviscid micropolar fluid can develop singularity in finite time is a challenging problem. For the 2D micropolar fluid with partial viscosity, the global regularity of the solution to the 2D micropolar fluid with zero angular viscosity has been established by Dong and Zhang in [8]. Xue in [9] established the global well-posedness to 2D micropolar fluid with zero kinetic viscosity or zero angular viscosity. There are more works for the 2D micropolar fluid in [10–13].

If the effect of micro-rotation field is negligible and $\chi = 0$, the system (1.2) reduces to the following famous magneto-hydrodynamic equations (MHD), which is written as

$$\begin{cases} \partial_t u + u\cdot\nabla u + \nabla\pi = \mu\Delta u + b\cdot\nabla b, \\ \partial_t b + u\cdot\nabla b = \nu\Delta b + b\cdot\nabla u, \\ \nabla\cdot u = \nabla\cdot b = 0. \end{cases} \tag{1.4}$$

Consider the 2D MHD equations with full viscosity, the existence and uniqueness of classical solution for the large initial data has been obtained in [14]. However, the global regularity issue of inviscid MHD equations is a challenging open problem. As an intermediate case, Cao and Wu established the global regularity of the 2D MHD equations with mixed partial dissipation and magnetic diffusion in [15]. Please see [16–18] to learn more results for 2D MHD equations with partial viscosity.

Motivated by their works in [15], our main goal in this paper is to establish the global regularity of 2D magnetic micropolar fluid flows with mixed partial viscosity, which can be expressed as follows

$$\begin{cases} \partial_t u + u\cdot\nabla u + \nabla\pi = \partial_{yy} u + b\cdot\nabla b + \nabla^{\perp} m, \\ \partial_t b + u\cdot\nabla b = \partial_{xx} b + b\cdot\nabla u, \\ m_t + u\cdot\nabla m = m_{xx} + \nabla\times u - 2m, \\ \nabla\cdot u = \nabla\cdot b = 0. \end{cases} \tag{1.5}$$

Without loss of generality, we take $\mu = \chi = \frac{1}{2}$ and $\nu = \kappa = 1$. Here, we impose the initial data as

$$u(x, y, 0) = u_0, \qquad b(x, y, 0) = b_0, \quad \text{and} \quad m(x, y, 0) = m_0. \tag{1.6}$$

Precisely, we have the following results.

Theorem 1.1. *Suppose that $u_0, b_0, m_0 \in H^2(\mathbb{R}^2)$ and $\operatorname{div} u_0 = \operatorname{div} b_0 = 0$, the Cauchy problem (1.5)–(1.6) has a unique global classical solution (u, b, m) with*

$$u, b, m \in L^{\infty}\left([0, T); H^2(\mathbb{R}^2)\right), \quad \omega_y, j_x, \nabla m_x \in L^2\left([0, T); H^1(\mathbb{R}^2)\right),$$

for any $T > 0$ independent of initial data, where $\omega = \nabla\times u$ and $j = \nabla\times b$ represent the vorticity and the current density, respectively.

Remark 1.1. Moreover, we consider the 2D anisotropic magneto-micropolar fluid flows with horizontal kinematic viscosity, vertical magnetic diffusion and vertical vortex viscosity,

$$\begin{cases} \partial_t u + u\cdot\nabla u + \nabla\pi = \partial_{xx} u + b\cdot\nabla b + \nabla^{\perp} m, \\ \partial_t b + u\cdot\nabla b = \partial_{yy} b + b\cdot\nabla u, \\ m_t + u\cdot\nabla m = m_{yy} + \nabla\times u - 2m, \\ \nabla\cdot u = \nabla\cdot b = 0. \end{cases} \tag{1.7}$$

Using the similar arguments in the proof of Theorem 1.1, we can also establish the global regularity for the system (1.7).