

Choosing the ART relaxation parameter for Clear-PEM 2D image reconstruction

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ABSTRACT

The Algebraic Reconstruction Technique (ART) is an iterative image reconstruction algorithm. During the development of the Clear-PEM device, a PET scanner designed for the evaluation of breast cancer, multiple tests were done in order to optimise the reconstruction process. The comparison between ART, MLEM and OSEM indicates that ART can perform faster and with better image quality than the other, most common algorithms. It is claimed in this paper that if ART's relaxation parameter is carefully adjusted to the reconstruction procedure it can produce high quality images in short computational time. This is confirmed by showing that with the relaxation parameter evolving as a logarithmic function, ART can match in terms of image quality and overcome in terms of computational time the performance of MLEM and OSEM algorithms. However, this study was performed only with simulated data and the level of noise with real data may be different.

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1. Introduction

The high incidence of breast cancer and the relative low specificity of the conventional detection methods (X-ray and ultrasound mammography) suggest the need for new imaging techniques. The Clear-PEM device, currently being developed for positron emission mammography (PEM) [1], answers this need by allowing the evaluation of early malign neoplasm in the breast and of ganglion loco-regional invasion.

In nuclear medicine, obtaining tomographic images from projections can be performed either by using analytical or

iterative reconstruction algorithms. There are two types of iterative algorithms: algebraic and statistical. The Algebraic Reconstruction Technique (ART) is in the aim of this study. This algorithm is the most commonly used image reconstruction algebraic iterative algorithm and is the earliest image reconstruction method from a large family of methods called constraint-satisfaction methods. It was developed by Kaczmarz to solve systems of linear equations and was firstly applied to medical image reconstruction by Herman [2]. Specifically, we aimed at a detailed study of the influence of the relaxation parameter over the performance of a ART algorithm when compared with two statistical iterative

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algorithms for 2D image reconstruction, MLEM (maximum-likelihood expectation-maximization) [3,4] and OSEM (ordered subsets expectation-maximization) [5], for the specific case of noisy data, acquired using planar detectors. This study also intends to shed some clues on finding a criterion for varying the relaxation parameter value along iteration number in order to maximise image quality using the shortest amount of time possible.

The task of recovering an object from its projections implies the construction of some specific variables. Let Y_i be the sum of all events detected along the i th line over the object, f_j the activity in the j th voxel of the object and a_{ij} the probability that annihilation in j is detected in a line of response (LOR) that contributes to the integral Y_i . In these conditions each line integral of the activity results from the sum (Eq. (1))

$$Y_i = \sum_j a_{ij} f_j \quad (1)$$

We can understand each measurement, Y_i as a hyperplane where the solution of f must lie. There are as many hyperplanes as projections and the solution of f must belong simultaneously to all hyperplanes, which means that the solution will lie on the intersection of all hyperplanes. In the presence of noiseless data, if we have at least the same number of projections and image pixels to calculate, the intersection of all hyperplanes will be a single point and the solution will be unique.

ART's iterative process starts with the definition of a first estimate of function f . This initial estimate will correspond to a vector in the space defined by the hyperplanes. The first iteration of the algorithm will correspond to the projection of this vector onto one of the hyperplanes, by determining the point on the hyperplane closest to the point of the estimate. This new point will then be used as an estimate for the next iteration that will consist in projecting it onto other hyperplane. This procedure is repeated for all hyperplanes, forcing the estimate to converge to the desired solution (Fig. 1).

The numerical expression that represents this operation is given by Eq. (2). In Eq. (2), k is the iteration number and a_{ij} is the model of the emission and detection processes corresponding to the probability that a detection Y_i had been originated from an event in pixel j

$$f_j^k = f_j^{k-1} + \frac{Y_i - \sum_l a_{il} f_j^{k-1}}{\sum_l a_{il}^2} a_{ij} \quad (2)$$

By multiplying the estimate resulting from the previous iteration by the system matrix, we are calculating the expected measurements from the estimated activity. After that, we compare these expected measurements with the real measurements, Y_i , using subtraction. The next step consists in multiplying the difference obtained by the respective system matrix element, performing the backprojection operation. Finally, we update the estimate from the previous iteration by adding the correction factor obtained from the backprojection operation.

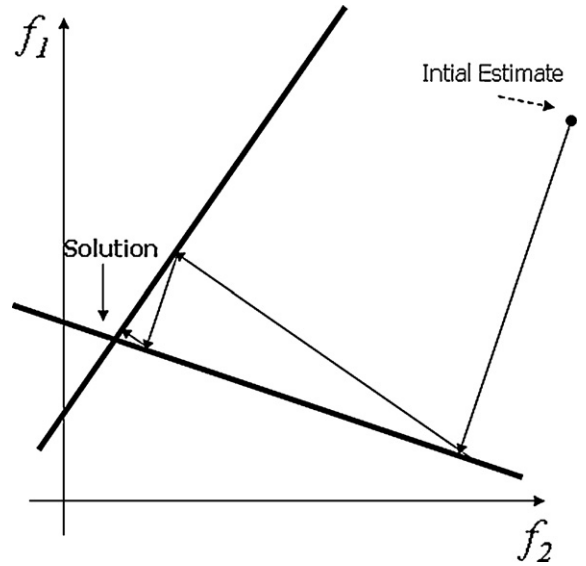


Fig. 1 – An inverse problem with two pixels with an unknown activity (f_1 and f_2) and two measurements (Y_1 and Y_2) defining two hyperplanes. The solution lies on the intersection of all hyperplanes (adapted from [14]).

If we analyse the inverse problem using a geometrical approach, we realise that convergence speed is highly dependent on the hyperplanes considered to find the solution. The obvious mathematical explanation is that the more perpendicular the two consecutive hyperplanes are, more new information we are adding to solve the problem, opposing to the situation where we are using very close hyperplanes, having both almost the same information and not contributing largely to the solution [6].

When noise is present in the acquired data, the assumption that all hyperplanes intersect in a single point is generally not true since each hyperplane is slightly shifted from its original position (Fig. 2 (left)). In this situation, multiple intersections corresponding to partial solutions will probably arise, i.e., solutions that satisfy part of the constraints but not all of them. This will be a problem to the iterative procedure since it will not converge to a unique solution, switching cyclically between partial solutions (Fig. 2 (middle)). This kind of behaviour can be limited by introducing a relaxation parameter (λ) in the algorithm expression used to update the solution f (Eq. (3))

$$f_j^k = f_j^{k-1} + \lambda^k \frac{Y_i - \sum_l a_{il} f_j^{k-1}}{\sum_l a_{il}^2} a_{ij} \quad (3)$$

In order to keep the result in the space between the different hyperplanes, minimising the distance to the different partial solutions, this relaxation parameter must have a value between 0 and 1, limiting the update process. By doing this, we prevent the algorithm from entering in a loop, forcing the convergence to a point somewhere in the middle of the partial solutions (Fig. 2 (right)).

The choice of the relaxation parameter value is critical to the quality and speed of image reconstruction using ART

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