Computers and Mathematics with Applications 69 (2015) 275–290

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Optimal boundary control for water hammer suppression in fluid transmission pipelines[☆]

Tehuan Chen [a], Zhigang Ren [a], Chao Xu [a,*], Ryan Loxton [b]

[a] State Key Laboratory of Industrial Control Technology and Institute of Cyber-Systems & Control, Zhejiang University, Hangzhou, Zhejiang 310027, China

[b] Department of Mathematics & Statistics, Curtin University, Perth, Western Australia 6845, Australia

ARTICLE INFO

Article history:
Received 15 April 2014
Received in revised form 24 September 2014
Accepted 10 November 2014
Available online 8 January 2015

Keywords:
Water hammer
Optimal boundary control
Method of lines
Hyperbolic partial differential equation
Control parameterization method

ABSTRACT

When fluid flow in a pipeline is suddenly halted, a pressure surge or wave is created within the pipeline. This phenomenon, called water hammer, can cause major damage to pipelines, including pipeline ruptures. In this paper, we model the problem of mitigating water hammer during valve closure by an optimal boundary control problem involving a nonlinear hyperbolic PDE system that describes the fluid flow along the pipeline. The control variable in this system represents the valve boundary actuation implemented at the pipeline terminus. To solve the boundary control problem, we first use the method of lines to obtain a finite-dimensional ODE model based on the original PDE system. Then, for the boundary control design, we apply the control parameterization method to obtain an approximate optimal parameter selection problem that can be solved using nonlinear optimization techniques such as Sequential Quadratic Programming (SQP). We conclude the paper with simulation results demonstrating the capability of optimal boundary control to significantly reduce flow fluctuation.

1. Introduction

Water hammer occurs when fluid moving through a pipeline is forced to suddenly stop or change direction. This sudden change in motion, which could be due to valve closure, pump failure, or unexpected pipeline damage, causes a pressure wave to propagate along the pipeline at high speed [1,2]. The wave speed can be over 1000 m/s, with significant pressure oscillation, often causing loud noises and serious damage [3]. In severe cases, water hammer may even cause the pipeline to rupture, resulting in slurry and water leakage (examples of pipeline rupture are shown in Fig. 1) [4]. Fluid pipeline failures due to water hammer effects are described in detail in [5,6].

The mathematical equations describing water hammer consist of hyperbolic or parabolic partial differential equations. Numerous methods for solving these equations, and thereby simulating water hammer, have been developed over the past forty years. These methods can be divided into three groups: analytical methods [7], graphical methods [8] and numerical methods [9]. The graphical and analytical methods are only applicable under various simplifying assumptions, and thus their value is limited in practical scenarios. In particular, the graphical and analytical methods cannot deal with the cavitation caused by negative pressure [10]. Numerical methods for simulating water hammer include the fluid–structure

☆ This work was partially supported by the National High Technology Research and Development Program of China through grant 2012AA041701 and the National Natural Science Foundation of China through grants F030119-61104048 and 61320106009.

* Corresponding author.
E-mail address: cxu@zju.edu.cn (C. Xu).

http://dx.doi.org/10.1016/j.camwa.2014.11.008
0898-1221/

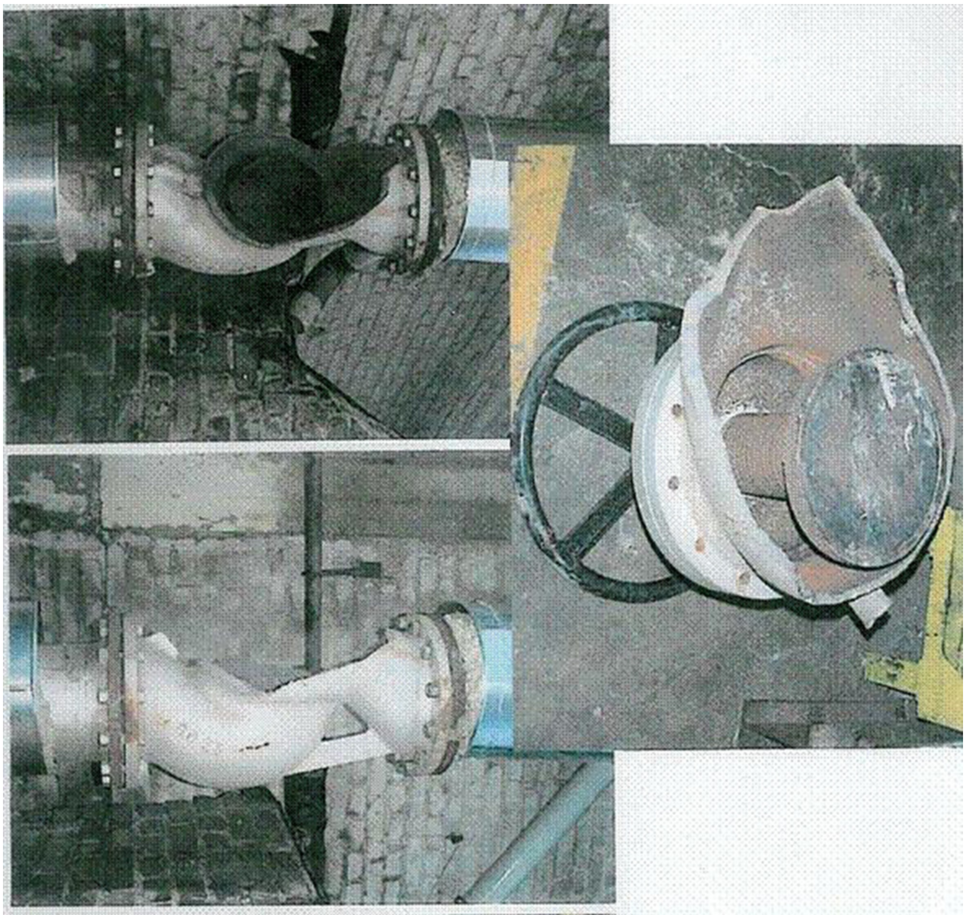

Fig. 1. Examples of pipeline damage caused by water hammer.
Source: http://traction.armintl.com/traction#/single&proj=Docs&rec=403&brief=n.

interaction method [11], the method of characteristics [12,13], the heterogeneous multiscale method [14], the finite volume method [15], and the wave plan method [16]. In this paper, we apply the method of lines [17,18] to approximate the water hammer PDEs by a system of ODEs. This approach enables the application of ODE optimal control techniques, for which there are many existing high-quality numerical algorithms, to determine optimal valve closure strategies to mitigate water hammer.

To protect a pipeline system from water hammer effects, various passive protection strategies can be employed. These include using special materials to reinforce the pipeline and installing special devices such as relief valves, air chambers, and surge tanks [19]. However, the success of these strategies depends heavily on the characteristics of the pipeline system and on the experience of the designer/operator [20]. Moreover, although passive protection strategies can act as a guard against water hammer, it is usually better to try and prevent water hammer from occurring in the first place. Hence, effective control strategies for valve closure are required to avoid the worst effects of water hammer, such as hazardous pipeline collapse.

The water hammer process involves nonlinearities and is non-uniform in space and time. Therefore, optimal flow control requires a forecasting model capable of predicting the non-uniform and unsteady water flow in space and time. Furthermore, due to flow nonlinearities, it is difficult to establish the relationship between the control action and the corresponding response in the hydrodynamic variables. Thus, effective valve control strategies are essential. Cao [21] used functional extremum theory and the Ritz method to design optimal rules for both velocity change and valve closure to minimize the peak pressure at the valve. Axworthy [22] developed a valve closure algorithm for node-based, graph-theoretic models that can be applied within a slow transient (rigid water column) pipeline network. Tian [23] investigated the optimum design of parallel pump feedwater systems in nuclear power plants to mitigate the potential damage caused by valve-induced water hammer. Feng [24] proposed an optimal control method for the regulation of multiple valves, focusing on the active causes of water hammer. Now, with the rapid development of modern control theory and numerical methodologies, advances in nonlinear optimization have made the solution of nonlinear flow control problems possible. Accordingly, in this paper, we propose an effective numerical approach to determine optimal boundary controls for valve closure in fluid pipelines.

The paper is organized as follows. In Section 2, we introduce a hyperbolic PDE system to describe the fluid flow dynamics in the pipeline, after which we propose an optimal control problem for water hammer suppression during valve closure. In