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Optimal piezoelectric control of a plate subject to time-dependent boundary moments and forcing function for vibration damping

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ABSTRACT

Active vibration control problem for a rectangular plate subject to moment boundary conditions and forcing function is solved by means of a maximum principle. The control is exercised by a patch actuator and the solution is formulated using an adjoint variable leading to a coupled boundary–initial-terminal value problem. The application of the maximum principle yields the optimal control expression as well as the explicit solution for a simply supported plate. The objective functional to be minimized is defined as a quadratic functional of displacement and velocity and also includes a penalty in terms of control voltage applied to the piezoelectric patch actuator. The penalty term limits the amount of control energy spent during the control process. Numerical results are presented to assess the effect of the optimal control algorithm.

1. Introduction

Plates are used in several engineering applications and exposed to a variety of dynamic loads under operational conditions. Such loads are often in the form of transient forces acting on the plate and/or time-dependent boundary conditions leading to forced vibrations. Many studies on the subject include [1–6] where rectangular plates undergoing forced vibrations have been investigated. Corresponding studies on the forced vibrations of beams with time dependent boundary conditions can be found in [7,8]. Present study is directed to the optimal control of rectangular plates to damp out vibrations caused by time-dependent transverse loads and moment boundary conditions using an open-loop controlled piezoelectric patch actuator. Damping out the excessive vibrations is of importance for a large number of structural components and especially important for aircraft wings and space crafts for operational reasons [9–11]. For this reason structures with surface bonded piezoelectric patch actuators have received considerable attention for the development of smart structures capable of damping out excessive vibrations. A review on vibration using smart material technology is given by [12].

Distributed piezoelectric actuators are used widely in structural applications for shape control, vibration suppression and acoustic control, and in particular active vibration control employing piezoelectric actuators has been studied extensively. One reason for this is that compared to other choices of actuators, piezoelectric devices are relatively light weight, can be easily embedded or bonded, have low energy requirements and smaller than traditional actuators used in vibration

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suppression. Recent studies on the vibration control of plates using piezoelectric actuators and sensors include [13–17]. In these studies several aspects of piezoelectric control have been studied including feedback control [14], constraints on control voltage [15], and open-loop control [17]. Optimal placement of patch actuators to improve control has been the subject of [18–20]. Numerical solutions of the active piezoelectric control problems have been obtained using finite element formulations in [21–23]. Piezoelectric control of functionally graded material (FGM) plates has been studied [24,25]. Use of piezoelectric actuators for controlling panel flutter has been the subject of [9,10]. In the above studies isotropic piezoelectric materials were employed as actuators and sensors while the orthotropic piezo-actuators can also be used for this purpose [26]. Special cases of vibration control due to thermal loads [27–30] and moving loads [31] using piezoelectric actuation were also studied. In the present study, the optimal vibration control problem for a simply supported rectangular plate subject to an external excitation and transient moment boundary conditions is formulated and solved with the control exercised by a piezo-patch actuator. The original contribution of the present paper to literature is that, in this paper, the effects of the external excitation and transient moment boundary conditions to a simply supported rectangular plate with the control exercised by a piezo-patch actuator are considered together to gain more realistic observation of the plate. Such cases of forced vibration arise in many practical situations due to external sources such as wind loading, machinery vibrations, and transient torsional loads on the boundaries. The open loop control law is derived by using a maximum principle which allows an analytical solution of the problem. The objective functional to be minimized is specified as the dynamic response of the plate which is defined as a quadratic functional of the displacement and velocity with the expenditure of control energy added as a penalty term. A control algorithm based on the maximum principle is formulated and shown to damp out the vibrations and minimize the dynamic response of the plate by using a minimum level of voltage. Numerical examples are given to demonstrate the effectiveness of the control algorithm with the plate subject to a forcing function or transient moment boundary conditions.

2. Problem formulation

We consider a simply supported rectangular plate, subject to the external excitation and moment boundary conditions, with piezoelectric patch actuator bonded on the plate. Initially, the plate is assumed to be undeformed. The control problem is aimed to suppress the vibrations induced by external excitation and moment boundary conditions using control voltages applied uniformly to the piezoelectric patch actuator. The equation of the motion of the plate shown in Fig. 1 can be expressed as follows [4];

$$\mu W_{TT} + D\nabla^4 W = F(X, Y, T) + \bar{V}(T)\nabla^2 \triangle[H(X, Y)] \tag{1}$$

where $W(X, Y, T)$ is the transversal displacement at $(X, Y, T) \in \check{Q} = \{(X, Y, T) : (X, Y) \in \check{S}, T \in (0, T_f)\}$, $\check{S} = (0, \ell) \times (0, \ell)$ is an open bounded set with sufficiently smooth boundary $\partial\check{S}$, T_f is the terminal time, $\mu > 0$ is the mass-per-unit area, $F(X, Y, T) = F_1(T)F_2(X, Y)$ is the forcing function with $F_2(X, Y)$ indicating the distribution of the force over the plate, $\bar{V}(T)$ is control voltage applied to the piezoelectric patch actuator, $D > 0$ is the flexural stiffness of the plate and

$$\nabla^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}.$$

In Eq. (1), the Heaviside function $H(X, Y)$ indicates the location of the patch actuator and is given by

$$\triangle[H(X, Y)] = [H(X - X_1) - H(X - X_2)] \times [H(Y - Y_1) - H(Y - Y_2)]$$

in which (X_1, Y_1) and (X_2, Y_2) are location of piezoelectric actuator. Eq. (1) is subject to the following boundary conditions

$$W(0, Y, T) = W(\ell, Y, T) = W(X, 0, T) = W(X, \ell, T) = 0, \tag{2}$$

$$W_{XX}(0, Y, T) = G_1(Y, T), \qquad W_{XX}(\ell, Y, T) = G_1(Y, T), \tag{3}$$

$$W_{YY}(X, 0, T) = H_1(X, T), \qquad W_{YY}(X, \ell, T) = H_2(X, T) \tag{4}$$

in which $G_1(Y, T)$ and $H_i(X, T)$ $(i = 1, 2)$ are defined as the moment of the friction force along the edges of the plate, and the initial conditions

$$W(X, Y, 0) = W_0(X, Y), \qquad W_T(X, Y, 0) = W_1(X, Y). \tag{5}$$

Let $L^2(\check{Q})$ denote the Hilbert space of real-valued square-integrable functions on the domain $\check{Q}$ in the Lebesgue sense with usual inner product and norm defined by

$$\langle \rho, \varrho \rangle_{\check{Q}} = \int_{\check{Q}} \rho(x, y, t)\varrho(x, y, t)d\check{Q}, \quad \|\rho\|^2 = \langle \rho, \rho \rangle,$$

respectively. Following conditions are imposed on the solution:

$$\frac{\partial^2 W}{\partial T^2}, \frac{\partial^n W}{\partial X^n}, \frac{\partial^n W}{\partial Y^n}, \frac{\partial^n W}{\partial X^i \partial Y^j} \in L^2(\check{Q}), \quad i, j = 0, 1, 2, \; n = 0, 1, \ldots, 4, \tag{6a}$$

$$W_0(X, Y) \in H_0^1(\check{S}), \qquad W_1(X, Y) \in L^2(\check{S}), \qquad \bar{V}(T) \in U_{ad}, \tag{6b}$$

$$\bar{U}_{ad} = \{\bar{V}(T) \mid \bar{V}(T) \in L^2(0, T_f), \; |\bar{V}(T)| \le M_0 < \infty\}, \tag{6c}$$

$$F(X, Y, T), \qquad G_1(Y, T) \quad \text{and} \quad H_i(X, T) \in L^2(\check{Q}) \quad (i = 1, 2). \tag{6d}$$