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Using the swarm intelligence algorithms in solution of the two-dimensional inverse Stefan problem

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ABSTRACT

In the paper a procedure for solving the two-dimensional inverse Stefan problem is presented. In considered problem the heat transfer coefficient is identified with the aid of known measurements of temperature in selected points of the region as the additional information. Direct Stefan problem is solved by using the alternating phase truncation method. Goal of the paper is to compare two swarm intelligence algorithms – the Ant Colony Optimization algorithm and the Artificial Bee Colony algorithm – applied for minimizing a functional expressing the error of approximate solution.

1. Introduction

Inverse problem is a task consisting in reconstruction of some parameters of a model on the basis of observed values. In case of physical processes, described by means of mathematical models, the inverse problems can be used, for example, for determining the values which cannot be directly measured or in the so called design problems in which the appropriate initial values of parameters should be determined for ensuring a required run of the process. Cost of applying the inverse problems in analysis of physical processes lies in the difficulties which may appear while solving this kind of problems. Analytic solution of an inverse problem not always exists, and even if it exists, it may be neither unique nor stable [1].

Most of the published works about the inverse Stefan problem concern the one-dimensional version of this problem (see [2–13], and references therein), whereas studies referring to the two-dimensional inverse Stefan problem are minor. In the first study related to the two-dimensional inverse design Stefan problem [14] the temperature at boundary of the domain was determined in form of a series of integrals based on the known interface position. Theoretical results concerning some selected cases of the inverse Stefan problem are included in papers [15–19].

In paper [3] the authors considered the one-dimensional one-phase inverse Stefan problem in which the distribution of temperature and the heat flux on one of the boundaries were determined in case when the interface location (defining the other boundary of the region) was given. The sought solution was approximated with the finite linear combination of functions satisfying the heat conduction equation and creating the complete family in space in which the solution was searched for. The problem was reduced to minimization of error under the given conditions (initial, boundary and Stefan conditions) realized by appropriate selection of coefficients of the mentioned linear combination. Extension of the applied method for the case of two-dimensional one-phase inverse Stefan problem was described in papers [20,21]. In [22] the two-dimensional one-phase inverse Stefan problem was treated as a problem of the non-linear approximation theory consisted in selection of a particular heat flux minimizing the functional defined as the norm of difference between the given interface position and the position reconstructed for the given heat flux. Ang with his collaborators considered in his papers the inverse Stefan prob-

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lem lying in determination of the heat flux on boundary of the region for which the interface takes the assumed location. The task was firstly transformed to the Volterra integral equation of the first kind and next, in result of regularization, to the equation of convolution type on the basis of which the authors estimated the error of regularized solution. The described method was adapted for two-dimensional case in papers [23–26]. In these papers the temperature [23,25], or relation between the heat flux and the temperature [26], was determined on one of boundaries of the region being the curvilinear quadrangle, one side of which was represented by the interface and the entire region was taken by the liquid phase. Task of calculating the temperature on boundary of the region in case of two-dimensional and two-phase problem was discussed in paper [24]. The two-dimensional problem was investigated as well in paper [27–29]. In these papers the heat flux or the temperature was determined on boundary of the domain for the known velocity of interface. For finding the solution the sensitivity coefficients and the finite-element method were used. In paper [29] the multidimensional case was studied on the way of minimization through the conjugate-gradient method in a finitely dimensional space. Next, the dynamic programming methods were used to solve the two-dimensional inverse Stefan problem in paper [28], whereas the method of fundamental solutions applied for solving the two-dimensional one- and two-phase design inverse Stefan problem was presented in papers [30,31].

Two-dimensional two-phase inverse Stefan problem was also considered by Grzymkowski and Słota in papers [32–35] and by Słota in papers [36–38]. In works [32,34] the solution was sought in the form of linear combination of functions satisfying the heat conduction equation and for calculating the coefficients of this combination the least square method was used on the way of minimizing the maximal defect in the initial–boundary data. The next method, investigated by Grzymkowski and Słota in papers [33,35], consisted in minimizing the functional, value of which represented the norm of difference between given positions of the moving interface and its positions reconstructed from the selected function describing the convective heat transfer coefficient. In papers [36–38], on the basis of given information about the measurements of temperature, the functional determining the error of approximate solution was constructed and for finding minimum of this functional the genetic algorithm was used. Examples of exploiting the evolutionary algorithms in solving some other inverse problems are presented in works [39–45].

The current paper is devoted to the two-dimensional inverse Stefan problem and taken task is to reconstruct the heat transfer coefficient on the basis of values of temperature measured in selected points of investigated region. For solving the direct Stefan problem we decided to apply the finite-difference method combined with the alternating phase truncation method [46,47,8,38].

On the ground of given information about the measurements of temperature a functional expressing the error of approximate solution is created. For minimizing this functional the appropriate optimization procedures must be applied. Most of the classical optimization algorithms require to satisfy various assumptions restricting their usage to the problems of specific kinds. Much more universal are the algorithms inspired by the nature, like for example the genetic algorithms or the algorithms based on the natural behaviors of insect swarms, the only condition of which is the existence of solution. In the present paper we intend to investigate the usefulness of Ant Colony Optimization (ACO) algorithm [48–50] and Artificial Bee Colony (ABC) algorithm [51–54]. These two algorithms are grounded on the behavior of societies of the insects searching for the food. Swarms of bees or ants, thanks to their numbers as well as the unique strategy of acting and sharing the pieces of information, can quickly find the optimal path leading to the source of food. The mentioned algorithms are characterized by high efficiency in solving the minimization tasks, especially in case of functions of several variables and with many local minima of similar values, as well as by the lack of conditions imposed on the discussed problem. This elasticity and universality of biologically inspired algorithms justify the idea of using them in the process of minimizing the functionals resulting from investigated problem. Application of optimization algorithms inspired by the nature in solving various kinds of inverse problems can be also found in papers [55–58]. In some earlier research Authors of the present papers have used also the classical minimization algorithms. Comparison of the classical Nelder–Mead algorithm and the genetic algorithm applied for solving the Stefan design problem is presented in paper [8]. Algorithms, used in the current paper, appeared to be equally precise as the genetic algorithm, but faster in working.

Classical minimization algorithms are characterized by the fast working, but from the other hand they are sensitive to the proper choice of the starting point, which may imply the possibility of getting stuck in the local minima. Whereas the biologically inspired algorithms are resistant to getting stuck in the local minima. Therefore, in order to speed up the running of procedures, we plan for the future to develop the hybrid algorithms, in which the biologically inspired algorithms will serve for determining the starting point (close to the sought global minimum) for the classical algorithm.

2. Two-dimensional problem

Let $\overline{\Omega} = [0, b] \times [0, d] \subset \mathbb{R}^2$ be the investigated region. Boundary of region $\Omega = \overline{\Omega} \times [0, t^*]$ is divided into following pieces (Fig. 1):

$$\begin{aligned}
\Gamma_0 &= \{(x, y, 0);\ x \in [0, b],\ y \in [0, d]\},\\
\Gamma_1 &= \left\{(0, y, t);\ y \in [0, d],\ t \in [0, t^*]\right\},\\
\Gamma_2 &= \left\{(x, 0, t);\ x \in [0, b],\ t \in [0, t^*]\right\},\\
\Gamma_3 &= \left\{(b, y, t);\ y \in [0, d],\ t \in [0, t^*]\right\},\\
\Gamma_4 &= \left\{(x, d, t);\ x \in [0, b],\ t \in [0, t^*]\right\},
\end{aligned}$$

for which the appropriate initial and boundary conditions are defined.