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Well-conditioned boundary integral equation formulations for the solution of high-frequency electromagnetic scattering problems

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ABSTRACT

We present several versions of Regularized Combined Field Integral Equation (CFIER) formulations for the solution of three dimensional frequency domain electromagnetic scattering problems with Perfectly Electric Conducting (PEC) boundary conditions. Just as in the Combined Field Integral Equations (CFIE), we seek the scattered fields in the form of a combined magnetic and electric dipole layer potentials that involves a composition of the latter type of boundary layers with regularizing operators. The regularizing operators are of two types: (1) modified versions of electric field integral operators with complex wavenumbers, and (2) principal symbols of those operators in the sense of pseudodifferential operators. We show that the boundary integral operators that enter these CFIER formulations are Fredholm of the second kind, and invertible with bounded inverses in the classical trace spaces of electromagnetic scattering problems. We present a spectral analysis of CFIER operators with regularizing operators that have purely imaginary wavenumbers for spherical geometries—we refer to these operators as Calderón–Ikawa CFIER. Under certain assumptions on the coupling constants and the absolute values of the imaginary wavenumbers of the regularizing operators, we show that the ensuing Calderón–Ikawa CFIER operators are coercive for spherical geometries. These properties allow us to derive wavenumber explicit bounds on the condition numbers of Calderón–Ikawa CFIER operators. When regularizing operators with complex wavenumbers with non-zero real parts are used—we refer to these operators as Calderón-Complex CFIER, we show numerical evidence that those complex wavenumbers can be selected in a manner that leads to CFIER formulations whose condition numbers can be bounded independently of frequency for spherical geometries. In addition, the Calderón-Complex CFIER operators possess excellent spectral properties in the high-frequency regime for both convex and non-convex scatterers. We provide numerical evidence that our solvers based on fast, high-order Nyström discretization of these equations converge in very small numbers of GMRES iterations, and the iteration counts are virtually independent of frequency for several smooth scatterers with slowly varying curvatures.

1. Introduction

The simulation of frequency domain electromagnetic wave scattering gives rise to a host of computational challenges that mostly result from oscillatory solutions, and ill-conditioning in the low and high-frequency regimes. Computational

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modeling of electromagnetic scattering has been attempted based on the classical Finite-Difference Time-Domain (FDTD) methods. However, algorithms based on the finite-difference or finite-element discretizations require discretization of unoccupied volumetric regions and give rise to numerical dispersion which is inevitably associated with numerical propagation of waves across large numbers of volumetric elements [1]. An important computational alternative to finite-difference and finite-element approaches is found in boundary integral methods. Numerical methods based on integral formulations of scattering problems enjoy a number of attractive properties as they formulate the problems on lower-dimensional, bounded computational domains and capture intrinsically the outgoing character of scattered waves. Thus, on account of the dimensional reduction and associated small discretizations (significantly smaller than the discretizations required by volumetric finite-element or finite-difference approximations), in conjunction with available *fast* solvers [2–12], numerical algorithms based on integral formulations, when applicable, can outperform their finite-element/difference counterparts. On account of this, and whenever possible, the simulation of high-frequency scattering problems relies almost exclusively on boundary integral equations based solvers. There has been significant recent progress on extending the range of high-frequency solvers that can be solved by boundary integral equation solvers, mostly in the case of scalar problems with Dirichlet boundary conditions. This was made possible by hybrid methods that incorporate the known oscillatory behavior of solutions of boundary integral equations at high frequencies in order to reduce drastically the number of unknowns—see the excellent review paper [13] for a full account of these methods.

While well-conditioned integral formulations for scalar problems with Dirichlet boundary conditions have been known and used for quite some time, that is not the case for electromagnetic problems. The scope of this paper is to address the question: what integral equations should one use for the efficient simulation of high-frequency frequency-domain electromagnetic scattering problems. The most widely used integral equation formulations for solution of frequency domain scattering problems from perfectly electric conducting (PEC) closed three-dimensional objects are the Combined Field Integral Equations (CFIE) formulations [14]. The CFIE are uniquely solvable throughout the frequency spectrum, yet the spectral properties of the boundary integral operators associated with the CFIE formulations are not particularly suited for Krylov-subspace iterative solvers such as GMRES [11,15]. This is attributed to the fact that the electric field (EFIE) operator, which is a portion of the CFIE, is a pseudodifferential operator of order 1 [16,17]—that is, asymptotically, the action of the operator in Fourier space amounts to multiplication by the Fourier-transform variable. Consistent with this fact, the eigenvalues of these operators accumulate at infinity, which causes the condition numbers of CFIE formulations to grow with the discretization size, a property that is shared by integral equations of the first kind. The lack of well conditioning of the operators in CFIE is exacerbated at high frequencies, a regime where CFIE require efficient preconditioners that should ideally control the amount of numerical work entailed by iterative solvers. In this regard, one possibility is to use algebraic preconditioners, typically based on multi-grid methods [18], or Frobenius norm minimizations and sparsification techniques [19]. However, the generic algebraic preconditioning strategies are not particularly geared towards wave scattering problems, and, in addition, they may encounter convergence breakdowns at higher frequencies that require large discretizations [20,21].

On the other hand, several alternative integral equation formulations for PEC scattering problems that possess good conditioning properties have been introduced in the literature in the past fifteen years [22–25,15,11,26–29]. Some of these formulations were devised to avoid the well-known "low-frequency breakdown" [30,28]. For instance, the current and charge integral equation formulation [29], although not Fredholm of the second kind, does not suffer from the low-frequency breakdown and has reasonable properties throughout the frequency range [31]. Another class of Fredholm boundary integral equations of the second kind for the solution of PEC electromagnetic scattering problems can be derived using generalized Debye sources [28]. Although these formulations targeted the low frequency case, their versions that use single layers with imaginary wavenumbers possess good condition numbers for higher frequencies for spherical scatterers [32].

Another wide class of formulations that is directly related to the present work can be viewed as Regularized Integral Equations as they typically involve using pseudoinverses/regularizers of the electric field integral operators to mollify the undesirable derivative-like effects of the latter operators. In the cases when the scattered electric fields are sought as linear combinations of magnetic and electric dipole distributions, the former acting on tangential densities while the latter acting on certain regularizing operators of the same tangential densities, the enforcement of the PEC boundary conditions leads to Regularized Combined Field Integral Equations (CFIER) or Generalized Combined Sources Integral Equations (GCSIE). In the case of smooth scatterers, the various regularizing operators proposed in the literature one the one hand (a) stabilize the leading order effect of the pseudodifferential operators of order 1 that enter CFIE, so that the integral operators in CFIER are compact perturbations of invertible diagonal matrix operators and (b) have certain coercivity properties that ensure the invertibility of the CFIER operators. One way to construct regularizing operators that achieve the objective (a) can be pursued in the framework of approximations of admittance/Dirichlet-to-Neumann operators (that is the operators that map the values of the vector product between the unit normal and the electric field on the surface of the scatterer to the value of the vector product between the unit normal and the magnetic field on the surface of the scatterer—see Section 5) [26,22,23,25] which can be connected to on-surface radiation conditions (OSRC) [33]. Another way to construct such operators is to start from Calderón's identities [15,24,27,11] that establish that the square of the electric field integral operator is a compact perturbation of the identity. All of these regularizing operators are either electric field integral operators, its vector single layer components, or their principal symbols in the sense of pseudodifferential operators. The regularizing operators that have property (a) can be modified to meet the requirement (b) either via quadratic partitions of unity [22,23] or by means of *complexification* of the wavenumber in the definition of electric field integral operators or its components