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Why dynamos are prone to reversals

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Abstract

In a recent paper [F. Stefani, G. Gerbeth. Asymmetric polarity reversals, bimodal field distribution, and coherence resonance in a spherically symmetric mean-field dynamo model. Phys. Rev. Lett. 94 (2005) 184506] it was shown that a simple mean-field dynamo model with a spherically symmetric helical turbulence parameter α can exhibit a number of features which are typical for Earth's magnetic field reversals. In particular, the model produces asymmetric reversals (with a slow decay of the dipole of one polarity and a fast recreation of the dipole with opposite polarity), a positive correlation of field strength and interval length, and a bimodal field distribution. All these features are attributable to the magnetic field dynamics in the vicinity of an exceptional point of the spectrum of the non-selfadjoint dynamo operator where two real eigenvalues coalesce and continue as a complex conjugated pair of eigenvalues. Usually, this exceptional point is associated with a nearby local maximum of the growth rate dependence on the magnetic Reynolds number. The negative slope of this curve between the local maximum and the exceptional point makes the system unstable and drives it to the exceptional point and beyond into the oscillatory branch where the sign change happens. A weakness of this reversal model is the apparent necessity to fine-tune the magnetic Reynolds number and/or the radial profile of α in order to adjust the operator spectrum in an appropriate way. In the present paper, it is shown that this fine-tuning is not necessary in the case of higher supercriticality of the dynamo. Numerical examples and physical arguments are compiled to show that, with increasing magnetic Reynolds number, there is strong tendency for the exceptional point and the associated local maximum to move close to the zero growth rate line where the indicated reversal scenario can be actualized. Although exemplified again by the spherically symmetric α^2 dynamo model, the main idea of this "self-tuning" mechanism of saturated dynamos into a reversal-prone state seems well transferable to other dynamos. As a consequence, reversing dynamos might be much more typical and may occur much more frequently in nature than what could be expected from a purely kinematic perspective.

Keywords: geomagnetic field; reversals; superchrons; superplumes

1. Introduction

The magnetic field of the Earth is known to undergo irregular reversals of its dipole part. The reversal rate is variable in the course of time: it was nearly zero in the Kiaman and the Cretaceous superchrons and is approximately 5 per My in the present [2].

Much effort has been devoted to identify typical characteristics of the reversal process. In particular, it was claimed that reversals may have an asymmetric, saw-toothed shape, with the field of one polarity decaying slowly and recreating rapidly with opposite polarity, possibly to quite high intensities [3–5].

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Another hypothesis concerns a correlation between the polarity interval length and the magnetic field intensity [6,7]. Associated with this, a causal connection of the Cretaceous superplume and superchron period has been discussed [8,9]. It was Vogt who first suggested a correlation between volcanism and reversal rate [10]. The general idea behind this is that superplumes give rise to an increased heat transport from the core mantle boundary to the Earth surface with the result of an increased dynamo strength due to a higher temperature gradient driving the outer core flow [11]. While this idea was soon generally accepted (with some counter-arguments regarding the involved time-scales [12]), quite contrary implications for the reversal frequency were drawn from it. The first "school", advocating a negative correlation of interval length and energy supply to the dynamo, goes back to Loper and McCartney [13]. The second school, suspecting long intervals for a strong dynamo, was motivated by various mean-field dynamo models, for which a transition from anharmonic oscillations to superchrons for increasing magnetic Reynolds number was observed [14].

A third, and still controversially discussed observation concerns the bimodal distribution of the Earth's virtual dipole moment (VDM) with two peaks at about 4×10^{22} A m^2 and at about twice that value [15–17].

For decades, it has been a challenge for dynamo theoreticians to explain reversals and their characteristics. It was considered a breakthrough when Glatzmaier and Roberts observed a reversal process in their fully coupled three-dimensional simulation of the geodynamo [18] (cf. also [19] for a recent overview). The strange thing with these simulations is that they reproduce many features of Earth magnetic fields, including reversals, quite well despite the fact that they are working in parameter regions far beyond those of the real Earth. This deficiency applies, in particular, to the Ekman and the magnetic Prandtl number. A way out of this dilemma may lie with a reliable sub-grid scale modeling [20]. In this respect one should also notice recent efforts to link direct numerical simulations and mean-field dynamo models [21,22].

This brings us back from expensive simulations to the complementary tradition of understanding reversals in terms of reduced dynamo models. A very simple approach in this direction is the celebrated Rikitake dynamo of two-coupled disk dynamos [23,24].

Another model was studied by Hoyng et al. [25–27]. A mean-field dynamo model is reduced to an equation system for the amplitudes of the non-periodic axisymmetric dipole mode and for one periodic overtone under the influence of stochastic forcing. This simple model, which produces sudden reversals and a Poissonian distribution of the interval time, has also been employed to simulate the phenomenon of stochastic resonance [28]. Stochastic resonance was made responsible in a former paper [29] for an apparent 100 ky periodicity in the interval length distribution [30] (note however, that this periodicity is not settled yet [31]). An essential ingredient of Hoyng's model to explain the correct reversal duration and the interval length consistently is the use of a large turbulent resistivity which is hardly justified. At least nothing of this has been seen in the recent liquid sodium dynamo experiments [32].

A further approach to understand reversals relies on the transition between non-oscillatory and oscillatory eigenmodes of the dynamo operator [33–35]. Those transition points, which have been found in many dynamo models [36,37], are well known in operator theory as spectral branch points—"exceptional points" of branching type of non-selfadjoint operators [38]. Such branch points are characterized not only by coalescing eigenvalues but also by a coalescence of two or more (geometric) eigenvectors and the formation of a non-diagonal Jordan block structure with associated vectors (algebraic eigenvectors) [39–41].[1] This is in contrast to "diabolical points" [45] which are exceptional points of an accidental crossing of two or more spectral branches with an unchanged diagonal block structure of the operator and without coalescing eigenvectors [38,40].

In a recent paper [1], we have analyzed the magnetic field dynamics in the vicinity of an exceptional point in more detail. Although the used model, a mean-field dynamo of the α^2 type with a supposed spherically symmetric helical turbulence parameter α, is certainly far beyond the reality of the Earth dynamo (owing, in particular, to the missing North–South-asymmetry of α) it exhibited all mentioned reversal features: asymmetry, a positive correlation of field strength and interval length, and bimodal field distribution.

All those features together were attributed to the very peculiar magnetic field dynamics in the vicinity

[1] We note that *selfadjoint* operators in Hilbert spaces (as they describe e.g. observables in closed quantum systems) have real spectra and a strictly diagonal spectral decomposition (a trivial Jordan block decomposition). Non-trivial Jordan block structures occur rather generically at spectral real-to-complex transition points of non-selfadjoint operators. Other physical systems (apart from the dynamo) described by non-selfadjoint operators with non-trivial Jordan block decompositions are, e.g., microwave resonators [42] as well as bianisotropic crystals and their optical singularities [43]. An extended discussion of the underlying mathematics can be found, e.g., in [44].