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RESEARCH PAPER

Distribution of thermodynamic variables inside extra-solar protoplanets formed via disk instability

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Abstract In this paper, distribution of thermodynamic variables inside extra-solar protoplanets in their initial stages, formed by gravitational instability, is presented. The case of conduction–radiation is considered regarding the transference of heat inside the protoplanets. The results are found to compare well with the ones obtained by other investigations.

1. Introduction

Both core accretion and disk instability advocated in the past can, in principle, form gas giant protoplanets. In the core accretion model, the heavy element core is formed by the accretion of planetesimals from the disk followed by further accretion of the surrounding gas (Pollack et al., 1996; Hubickyj et al., 2005). This mechanism has been adopted as the main theory of planetary formation both in our solar system and elsewhere. With the difficulties encountered with the core accretion models, the alternative theory with disk instability and the gravitational collapse of an unsegregated protoplanet which was in vogue in the 1970s when a great deal of now forgotten work was carried out has been reformulated with fragmentation from massive protoplanetary disks and has been advanced through the work of many authors (see e.g., Cha and Nayakshin, 2011; Nayakshin, 2010; Boley et al., 2010). Though some investigations argued that disk instabilities are unable to lead to the formation of self-gravitating, dense clumps (e.g., Pickett et al., 2000; Cai et al., 2006a,b; Boley et al., 2007a,b), the idea is believed to be the promising route for the rapid formation of giant planets in our solar system and elsewhere. Despite substantial study and progress in recent decades, the initial structures of isolated gaseous giant protoplanets formed via disk instability are still unknown and different models predict different initial characteristics (Helled and Schubert, 2008). As for example, the investigation of Nayakshin (2010) predicted colder protoplanets than the ones found in Helled and Schubert (2008) and Mayer et al. (2002, 2004) predicted denser and hotter protoplanets than the ones predicted by Boss (1997, 2007). Boss (1997) in his simulation assumed an initial protoplanet to be fully radiative, Helled and Schubert (2008) found such protoplanets to be fully convective with a thin outer radiative zone, while Paul et al. (2012) and Senthilkumar and Paul (2012) investigated the initial configurations of protoplanets assuming them to be fully convective.

In this paper we intend to determine the internal configuration of protoplanets formed by disk instability assuming the

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mode of transference of heat inside to be conductive-radiative and intend to show how they compare the results obtained through different approaches.

The rest of the paper is organized as follows. Section 2 deals with theoretical foundation of the problem. A detailed procedure of numerical approach for the solution is presented in Section 3. Result, discussion and conclusion are given in Section 4.

2. Theoretical foundation

2.1. Energy balance

Our model assumes a non-rotating, non-magnetic spherical giant gaseous object of solar composition in the mass range 0.3–10 M_J, where M_J is the mass of Jupiter. The choice of the mass range is because it covers most of the observed mass range of extrasolar giant planets (see, e.g., Helled and Schubert, 2008). The object is assumed to be in a steady state of quasi-static equilibrium in which the ideal gas law holds well. For heat transfer inside such an object, we consider the conductive–radiative case. We follow Bohm-Vitense (1997) for heat flux in the conductive–radiative heat transport in which the formulation states that the total heat flux in which both conduction and radiation play their role in transference of heat being given by

$$F(r) = 4\pi r^2\left(-\frac{16}{3\overline{K}}\sigma T^3\frac{dT}{dr}\right) \tag{1}$$

$$\text{with}\quad \frac{1}{\overline{K}} = \frac{1}{K_{cm}} + \frac{1}{K_{hc}}, \tag{2}$$

where σ is the Stefan–Boltzmann constant and η is the thermal conductivity of the gas and K_{cm} and $K_{hc} = 16\sigma T^3/(3\eta)$ are the radiative and conductive absorption coefficients respectively.

In a protoplanet, the source of energy being gravitational, some energy will be released due to its quasi-static contraction. Half of this released energy is used to raise the internal temperature and the other half goes through radiation. However, the system is in a steady state, so no heat will go into raising the temperature. Therefore, all the energy released will be available for energy flux. If we consider a spherical surface of radius r inside a protoplanet of radius R, the amount of energy available as the heat flux through the sphere of radius r is given by

$$F(r) = -\frac{dE(r)}{dt}, \tag{3}$$

where $E(r)$ is the total energy of the system of radius r and is given by

$E(r) = -\tau\frac{GM^2(r)}{r}$, where τ is a constant of order unity whose value depends on the internal structure of the system, G is the universal gravitational constant and $M(r)$ is the mass interior to a radius r.

Since $M(r)$ remains constant during contraction, therefore, with $E(r)$ Eq. (3) can be written as

$$F(r) = \tau\frac{GM^2(r)}{r^2}\frac{dr}{dt}. \tag{4}$$

For uniform contraction the Eq. (4) can be written as (see Paul et al., 2008)

$$F(r) = \frac{C_R}{R}\frac{GM^2(r)}{r}, \tag{5}$$

where C_R is an unknown constant. We shall consider this constant as a free parameter.

From Eqs. (1 and 5) with the help of Eq. (2), we get

$$-\frac{16}{3}\sigma\, T^3\frac{dT}{dR}\left(\frac{1}{K_{cm}} + \frac{1}{K_{hc}}\right) = \frac{C_R}{4\pi R}\frac{GM^2(r)}{r^3}.$$

Substituting for K_{hc}, we have

$$\left(\frac{16\sigma\ T^3(r)}{3K_{cm}} + \eta\right)\frac{dT(r)}{dr} = -C_R\frac{GM^2(r)}{4\pi R r^3}. \tag{6}$$

But $K_{cm} = nK_{at}$ (Bohm-Vitense, 1997), where n is the number of particles per unit volume and K_{at} is the absorption cross section of each particle. It is found that K_{at} is roughly equal to 2×10^{-24} cm^2 (Bohm-Vitense, 1997). With this value K_{cm} becomes

$$K_{cm} \approx \frac{2\times10^{-24}\rho(r)}{H},$$

where H is the mass of a hydrogen atom.

Substituting this value of K_{cm} in Eq. (6), we have the conductive–radiative flux in the form

$$\left(\frac{8\sigma H}{3\times10^{-24}}\frac{T^3(r)}{\rho(r)} + \eta\right)\frac{dT(r)}{dr} = -\frac{C_R}{4\pi R}\frac{GM^2(r)}{r^3}. \tag{7}$$

2.2. Protoplanetary structure

If the energy equation is given by (7), then the structure of the protoplanets can be given by the following set of equations:

The equation of hydrostatic equilibrium,

$$\frac{dP(r)}{dr} = -\frac{GM(r)}{r^2}\rho(r). \tag{8}$$

The equation of conservation of mass,

$$\frac{dM(r)}{dr} = 4\pi r^2\rho(r). \tag{9}$$

The equation of conductive–radiative heat flux,

$$\left(\frac{8\sigma H}{3\times10^{-24}}\frac{T^3\ (r)}{\rho(r)} + \eta\right)\frac{dT(r)}{dr} = -\frac{C_R}{4\pi R}\frac{GM^2(r)}{r^3}. \tag{10}$$

The gas law,

$$P(r) = \frac{k}{\mu H}\rho(r)T(r). \tag{11}$$

2.3. Boundary conditions

Considering a sphere of infinitesimal radius r at the center, we find that $M(r) = 4\pi r^3\rho/3$. Since we may treat ρ sensibly constant in this sphere, then as $r\to0$, $M(r)\to0$, ρ remains finite as $r\to0$. It is also clear that $M(r) = M$ at the surface, i.e., at $r = R$. The protoplanets having cold origin must have a low surface temperature. In the first approximation we assume that the surface temperature is zero. The mass of the atmosphere of a protoplanet is just a minute fraction of its total mass, so we may take the pressure on its surface as