Computers and Mathematics with Applications 65 (2013) 901–913

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Simulation of floating bodies with the lattice Boltzmann method

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ARTICLE INFO

Keywords:
Lattice Boltzmann
Free surface
Interface capturing
Particulate flow
Floating body
Fluid–structure interaction

ABSTRACT

This paper is devoted to the simulation of floating rigid bodies in free surface flows. For that, a lattice Boltzmann based model for liquid–gas–solid flows is presented. The approach is built upon previous work for the simulation of liquid–solid particle suspensions on the one hand, and on an interface-capturing technique for liquid–gas free surface flows on the other. The *incompressible* liquid flow is approximated by a lattice Boltzmann scheme, while the dynamics of the *compressible* gas are neglected. We show how the particle model and the interface capturing technique can be combined by a novel set of dynamic cell conversion rules. We also evaluate the behaviour of the free surface – particle interaction in simulations. One test case is the rotational stability of non-spherical rigid bodies floating on a plane water surface – a classical hydrostatic problem known from naval architecture. We show the consistency of our method in this kind of flows and obtain convergence towards the ideal solution for the heeling stability of a floating box.

1. Introduction

The lattice Boltzmann method (LBM) has become a popular alternative for complex flow simulations [1]. Its application to particle suspensions has been propelled in a significant part by the work of Ladd et al. [2,3] and Aidun et al. [4–6]. Based on the approach of the so-called *momentum exchange method*, it is possible to compute the hydromechanical stresses on the surface of fully resolved solid particles directly from the boundary conditions. In this paper, this fluid–solid coupling approach is extended to liquid–gas free surface flows, i.e., the problem of solid bodies moving freely within such a flow of two immiscible fluids, where the dynamics of one phase (the gas phase) can be regarded negligible. We use the free surface model of [7,8] to simulate a liquid phase in interaction with a gas by means of a *volume of fluids* approach and a special kinematic free surface boundary condition. Here, the interface of the two phases is assumed sharp enough to be modeled by a locally defined non-diffusive boundary layer. This boundary layer is updated dynamically according to the liquid advection based on a set of cell conversion rules.

This paper proposes a unification of the update rules of the free surface model with those of the particulate flow model. As described in [9], the resulting scheme allows full freedom of motion of the solid bodies in the flow, which can be calculated according to rigid body physics. We demonstrate the consistency of the combined liquid–gas–solid method by means of a simple advection test with a floating object in a stratified free surface channel flow, and discuss the primary sources of error in the dynamic boundary handling with particles in motion.

We further apply our method to the problem of rotational stability of rigid floating objects. This kind of hydromechanical problems typically emerges in marine engineering, where the floating stability of offshore structures is of concern [10,11]. This idea of evaluating the simulated floating stability of rigid bodies is inspired by Fekken [12], where it is proposed as a test

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doi:10.1016/j.camwa.2012.09.012

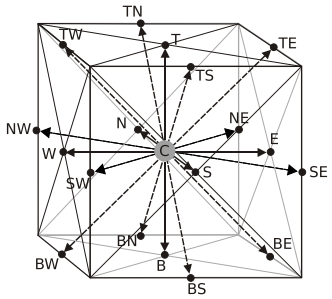

Fig. 1. D3Q19 stencil. The weights are $w_0 = 1/3$ for C, $w_1, \ldots, w_6 = 1/18$ for W, E, N, S, T, B, and $w_7, \ldots, w_{18} = 1/36$ for $TW, TE, TN, TS, NW, NE, SW, SE, BW, BE, BN, BS$.

case for a Navier–Stokes based simulator originally developed for the estimation of "green water" loads on ship decks [13]. To the best of our knowledge, the only approach to handle similar hydromechanic problems by means of lattice Boltzmann is the one by Janßen [14]. Because of the static nature of these problems, numerical issues arising from hydro-dynamic effects can be widely discarded, which makes them well-suited for the verification of the force calculations that are involved. In addition to that they provide a possibility to check the convergence of the simulated liquid–gas–solid systems into a state of equilibrium. We succeed in showing convergence in simulations, provided that adequate spatial resolutions are chosen. For the special problem of floating stability of cuboid structures, convergence of the numerical simulations toward the analytical model is obtained.

2. Method

2.1. Isothermal D3Q19 lattice BGK method

We assume the *D3Q19* lattice model for 3-dimensional flows [15–17], with a set of $N = 19$ discrete *lattice velocities* $\vec{c}_i$ ($i = 0, \ldots, N-1$). All of the calculations of Section 3 have been done with this model; but the techniques described in this article are general enough to be applied with other cubic lattice models. For the theoretical considerations in this section, however, we will often fall back implicitly to the native *D2Q9* model, as a 2-dimensional setting simplifies explanations and figures. The lattice velocities $\vec{c}_i$ (also called *lattice directions* or *lattice links*) with their respective weights w_i ($i = 0, \ldots, N-1$), as shown in Fig. 1, are $(0, 0, 0)$ for C, $(\pm1, 0, 0)$, $(0, \pm1, 0)$, $(0, 0, \pm1)$ for W, E, N, S, T, B, and $(\pm1, \pm1, 0)$, $(0, \pm1, \pm1)$, $(\pm1, 0, \pm1)$ for $TW, TE, TN, TS, NW, NE, SW, SE, BW, BE, BN, BS$. By $\bar{i}$ we denote the index of the lattice velocity $\vec{c}_{\bar{i}}$ with $\vec{c}_{\bar{i}} = -\vec{c}_i$. Let s_x, s_y, s_z be positive real numbers that are integer multiples of the *spatial resolution* δ_x. The domain $[0, s_x] \times [0, s_y] \times [0, s_z]$ is divided into *cells*, i.e., cubic volumes of length δ_x, which yields a computation domain of $l_x \times l_y \times l_z$ ($l_i := s_i/\delta_i$, $i \in \{x, y, z\}$), discrete lattice cells. Spatial quantities like s_x, s_y, s_z and δ_x are commonly given in a certain unit of length (e.g., metres). However, when dealing with LBM specific computations, dimensionless lattice coordinates are used: Spatial coordinates are thus given in the following as multiples of δ_x. By speaking of a cell (x, y, z), where x, y and z are positive integer numbers, we mean the lattice cell with respective volume $[x, x+1] \times [y, y+1] \times [z, z+1]$ in the lattice. We refer to the point $(x + 0.5, y + 0.5, z + 0.5)$ as the *cell center*. For each lattice direction $i = 0, \ldots, N-1$ we denote with $f_i(\vec{x}, t)$ the *particle distribution function* (PDF) of the direction $\vec{c}_i$ in cell $\vec{x}$ and of time step t.

The lattice BGK propagation scheme can be derived from the classic Boltzmann equation with the collision operator substituted by the *Bhatnagar Gross Krook* (BGK) operator [18–20]. Including an external force term F_i, the lattice BGK (LBGK) equation reads

$$f_i(\vec{x} + \delta_t \vec{c}_i, t + \delta_t) - f_i(\vec{x}, t) = -\frac{1}{\tau}\left[f_i(\vec{x}, t) + f_{eq,i}\left(\rho(\vec{x}, t), \vec{u}(\vec{x}, t)\right)\right] - \delta_t F_i. \tag{1}$$

Here, τ is the dimensionless relaxation time and related to the *kinematic viscosity* ν by $\tau = (\nu + 1/2c_s^2\delta_t)/(c_s^2\delta_t)$. The lattice *speed of sound* c_s is a model-dependent constant. In our case, we have $c_s = 1/\sqrt{3}$. The equilibrium function is therewith given as a so-called low Mach number expansion of the Maxwell distribution function,

$$f_{eq,i}(\rho, \vec{u}) = \rho w_i \left[1 + \frac{\vec{c}_i{}^T\vec{u}}{c_s^2} + \frac{(\vec{c}_i{}^T\vec{u})^2}{2c_s^4} - \frac{\vec{u}^T\vec{u}}{2c_s^2}\right], \tag{2}$$

see, e.g., [20]. Eq. (2) is valid for small flow velocities, where the constraint $\mathrm{Ma} := \sqrt{\vec{u}^T\vec{u}}/c_s \ll 1$ holds. The external force term F_i can be used to represent gravitation (expressed as acceleration $\vec{a}$) in simulations [21,22], with

$$F_i = w_i\rho\left(\frac{\vec{c}_i - \vec{u}}{c_s^2} + \frac{\vec{c}_i{}^T\vec{u}}{2c_s^4}\right)\vec{a}.$$