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Comparison of thermodynamics solvers in the polythermal ice sheet model SICOPOLIS

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A B S T R A C T

In order to model the thermal structure of polythermal ice sheets accurately, energy-conserving schemes and correct tracking of the cold–temperate transition surface (CTS) are necessary. We compare four different thermodynamics solvers in the ice sheet model SICOPOLIS. Two exist already, namely a two-layer polythermal scheme (POLY) and a single-phase cold-ice scheme (COLD), while the other two are newly-implemented, one-layer enthalpy schemes, namely a conventional scheme (ENTC) and a melting-CTS scheme (ENTM). The comparison uses scenarios of the EISMINT Phase 2 Simplified Geometry Experiments (Payne et al., 2000, J. Glaciol. 46, 227–238). The POLY scheme is used as a reference against which the performance of the other schemes is tested. Both the COLD scheme and the ENTC scheme fail to produce a continuous temperature gradient across the CTS, which is explicitly enforced by the ENTM scheme. ENTM is more precise than ENTC for determining the position of the CTS, while the performance of both schemes is good for the temperature/water-content profiles in the entire ice column. Therefore, the one-layer enthalpy schemes ENTC and ENTM are viable, easier implementable alternatives to the POLY scheme with its need to handle two different numerical domains for cold and temperate ice.

1. Introduction

Many glaciers and ice sheets are polythermal with disjoint cold and temperate domains, separated by the cold–temperate transition surface (CTS) (Blatter and Hutter, 1991). Both the Greenland and Antarctic ice sheets are Canadian-type polythermal, that is, they are mainly cold, except for distributed temperate layers at the base where strain heating is largest and where there is a geothermal contribution. It is thus important to model the thermodynamics of ice sheets correctly by distinguishing both domains and accounting for the transition conditions between them.

Various methods allow one to model the thermodynamic conditions in ice sheets. Thus far, SICOPOLIS (SImulation COde for POLythermal Ice Sheets; e.g., Greve, 1997b; Sato and Greve, 2012; Greve and Herzfeld, 2013; URL www.sicopolis.net) is the only three-dimensional ice sheet model that employs the *polythermal two-layer scheme*. In this method, the temperature and water-content fields in the two domains, cold and temperate ice, are computed on separate numerical domains, and the transition conditions at the CTS are used to track its position.

In most older ice sheet models (e.g., Huybrechts, 1990; Calov and Hutter, 1996; Payne and Dongelmans, 1997; Ritz et al., 1997), the *cold-ice method* was applied by resetting any computed temperatures that exceed the local pressure melting point to the local pressure melting point. While very simple, this means that energy is lost, and the water content in the temperate layer as well as the transition conditions at the CTS are ignored. The cold-ice method has, however, always been available in SICOPOLIS as an alternative to the polythermal two-layer method.

Aschwanden et al. (2012) introduced a new, enthalpy-based approach for ice sheet thermodynamics. In this method, the thermodynamic fields of temperature in cold ice and water content in temperate ice are replaced by one common thermodynamic field, enthalpy,[1] for both domains, and only one common field equation must be solved. However, the Stefan-type energy- and mass-flux matching conditions at the CTS, which are important for determining its position (Greve, 1997a), are not included explicitly in the formulation of the enthalpy scheme by Aschwanden et al. (2012).

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[1] Owing to the incompressibility of ice, the enthalpy is identical to the internal energy.

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Following the terminology of Blatter and Greve (2015), we refer to it as the *conventional one-layer enthalpy scheme*. This scheme has already been used in a number of ice sheet and glacier models (Brinkerhoff and Johnson, 2013; Golledge et al., 2013; Seroussi et al., 2013; Wilson and Flowers, 2013; Gilbert et al., 2014; Kleiner et al., 2015).

Two different conditions of the CTS must be distinguished. For melting conditions, cold ice flows across the CTS into the temperate layer, where water starts to accumulate due to strain heating along trajectories. The opposite situation, freezing conditions, occurs further downstream, where temperate ice flows across the CTS into the cold domain and the accumulated water content freezes out, releasing latent heat. For melting conditions, the temperature gradient and the water content are continuous across the CTS, while for freezing conditions, discontinuities of these quantities occur (Greve, 1997a). Since the CTS tends to be rather steep near the terminus, only a small area of the CTS is freezing, and therefore, in results of ice sheet models, freezing conditions usually only occur at very few isolated grid points (Greve, 1997b).

Kleiner et al. (2015) tested the implementation of the conventional enthalpy scheme for a Canadian-type parallel-sided slab in one finite-difference and two finite-element ice sheet models (TIM-FD[3], ISSM, COMice). Blatter and Greve (2015) compared the performance of four different versions of the enthalpy scheme for a parallel-sided slab with a custom-designed finite-difference program. Besides the conventional enthalpy scheme, they considered the *two-layer front-tracking enthalpy scheme* (an enthalpy version of the polythermal two-layer scheme mentioned above), the *one-layer melting-CTS enthalpy scheme* and the *one-layer freezing-CTS enthalpy scheme*. In the two latter schemes, explicit tracking of the melting or freezing CTS, based on the respective transition conditions at the CTS, has been added to the conventional enthalpy scheme. An important finding of these works was that the conventional one-layer enthalpy scheme can produce correct solutions for melting conditions at the CTS, provided that the numerical handling of the discontinuity of the enthalpy diffusivity across the CTS is done carefully. However, especially for finite-difference techniques, Blatter and Greve (2015) concluded that it is safer to use the one-layer melting-CTS enthalpy scheme, which enforces the transition conditions explicitly. For freezing conditions, the conventional one-layer enthalpy scheme fails because it cannot handle the associated discontinuities of the thermodynamic fields, and it is thus imperative to enforce the transition conditions at the CTS explicitly, as it is done in the one-layer freezing-CTS enthalpy scheme.

For this study, in addition to the previously existing polythermal two-layer and cold-ice schemes, we have implemented the conventional one-layer enthalpy scheme and the one-layer melting-CTS enthalpy scheme in the SICOPOLIS model. For the reason given above, freezing conditions are not considered here. We attempt to test and verify these four schemes in SICOPOLIS, and in particular to test how the various schemes handle the melting CTS between cold and temperate ice for Canadian-type polythermal situations in ice sheets. Based on the results of Blatter and Greve (2015), we consider the polythermal two-layer scheme to be the most reliable method and thus use its results as benchmark solutions. In Sections 2 and 3 we give an overview of the theory of ice-sheet thermodynamics and describe the implementation of the various schemes in SICOPOLIS. Section 4 gives the set-up of the scenarios derived from the suite of EISMINT (European Ice Sheet Modeling INiTiative) Phase 2 Simplified Geometry Experiments (Payne et al., 2000) used for this study. In Section 5 we discuss the results, focusing on the simulated thickness of the temperate ice layer. Section 6 concludes the paper.

2. Outline of ice-sheet thermodynamics

2.1. Standard polythermal thermodynamics

The standard description of the thermodynamics of polythermal ice masses, for which we follow largely Greve (1997a), is based on the fields of absolute temperature T in cold ice and water content W in temperate ice. The evolution equation for temperature in cold ice is given by

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\mathrm{grad}T = \frac{1}{\rho c}\frac{\partial}{\partial z}\left(\kappa\frac{\partial T}{\partial z}\right) + \frac{Q}{\rho c}, \tag{1}$$

where t denotes time, z the vertical spatial coordinate, $\mathbf{v}$ the three-dimensional velocity vector, $\rho = 910\ \mathrm{kg\ m^{-3}}$ the ice density, κ the temperature-dependent heat conductivity of cold ice and c the temperature-dependent heat capacity of cold ice. Also, $Q = \mathrm{tr}(\mathbf{t}\cdot\mathbf{D})$ is the volumetric strain heating, where $\mathbf{t}$ is the Cauchy stress tensor, $\mathbf{D}$ the strain-rate tensor, the middle dot $(\cdot)$ denotes tensor contraction and tr the trace of a tensor. Horizontal diffusion terms have been neglected, which can be justified by scaling arguments making use of the shallowness of ice sheets (e.g. Greve and Blatter, 2009).

Similar to Eq. (1), the evolution equation for water content in temperate ice reads

$$\begin{aligned}\frac{\partial W}{\partial t} + \mathbf{v}\cdot\mathrm{grad}W &= \frac{1}{\rho}\frac{\partial}{\partial z}\left(\nu\frac{\partial W}{\partial z}\right) + \frac{Q}{\rho L} \\ &\quad - \frac{c}{L}\left(\frac{\partial T_\mathrm{m}}{\partial t} + \mathbf{v}\cdot\mathrm{grad}T_\mathrm{m}\right) + \frac{1}{\rho L}\frac{\partial}{\partial z}\left(\kappa\frac{\partial T_\mathrm{m}}{\partial z}\right),\end{aligned} \tag{2}$$

where ν is the water diffusivity in temperate ice (assumed to be constant) and $L = 3.35\times10^{5}\ \mathrm{J\ kg^{-1}}$ the latent heat of fusion. The very small terms in the second line of the equation arise from the fact that the temperature in temperate ice is not constant, but equal to the melting temperature T_m that depends on the local pressure p,

$$T_\mathrm{m}(p) = T_0 - \beta p, \tag{3}$$

where $T_0 = 273.15$ K is the reference temperature and $\beta = 9.8\times10^{-8}\ \mathrm{K\ Pa^{-1}}$ the Clausius-Clapeyron constant for air-saturated glacier ice (Hooke, 2005). As in Eq. (1), horizontal diffusion terms have been neglected in the evolution equation (2).

As already mentioned in Sect. 1, for melting conditions at the CTS, the temperature gradient and the water content must be continuous across the CTS (Greve, 1997a). If we mark values at the cold side of the CTS by plus $(+)$ superscripts, values at the temperate side by minus $(-)$ superscripts, and denote the normal unit vector pointing into the cold side by $\mathbf{n}$, this reads

$$\mathrm{grad}T^{+}\cdot\mathbf{n} = \mathrm{grad}T_\mathrm{m}^{-}\cdot\mathbf{n} \tag{4}$$

and

$$W^{+} = W^{-} = 0. \tag{5}$$

For freezing conditions, the situation is more complicated in that the temperature gradient and the water content are in general discontinuous across the CTS (Blatter and Hutter, 1991; Greve, 1997a). However, as already mentioned in Sect. 1, this situation usually occurs only on very small parts of the CTS, and therefore we do not consider freezing conditions in this study.

At the ice surface, we prescribe the surface temperature as a Dirichlet-type boundary condition. At the ice base, three different cases must be distinguished. For a cold base, the geothermal heat