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Topology optimisation of repairable flow networks for a maximum average availability

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ABSTRACT

We state and prove a theorem regarding the average production availability of a repairable flow network, composed of independently working edges, whose failures follow a homogeneous Poisson process. The average production availability is equal to the average of the maximum output flow rates on demand from the network, calculated after removing the separate edges with probabilities equal to the edges unavailabilities. This result creates the basis of extremely fast solvers for the production availability of complex repairable networks, the running time of which is independent of the length of the operational interval, the failure frequencies, or the lengths of the downtimes for repair. The computational speed of the production availability solver has been extended further by a new algorithm for maximising the output flow in a network after the removal of several edges, which does not require determining the feasible edge flows in the network. The algorithm for maximising the network flow is based on a new theorem, referred to as 'the maximum flow after edge failures theorem', stated and proved for the first time.

Finally, unlike heuristic optimisation algorithms, the proposed algorithm for a topology optimisation of the network always determines the optimal solution.

The high computational speed of the developed production availability solver created the possibility for embedding it in simulation loops, performing a topology optimisation of large and complex repairable networks, aimed at attaining a maximum average availability within a specified budget for building the network. An exact optimisation method has been proposed, based on pruning the full-complexity network by using the branch and bound method as a way of exploring possible network topologies. This makes the proposed algorithm much more efficient, compared to an algorithm implementing a full exhaustive search. In addition, the proposed method produces an optimal solution compared to heuristic optimisation methods.

The application of the bound and branch method is possible because of the monotonic dependence of the production availability on the number of the edges pruned from the full-complexity network.

1. Introduction

Most real flow networks (e.g., oil and gas production networks, power distribution networks, computer networks) are, in effect, repairable flow networks. Indeed, after the failure of a component or section in a production flow line, computer network, power supply system, or water supply system, a repair is initiated, and, after a particular downtime, the component/section is returned to operation. An essential feature of repairable flow networks is that a repair of failed components is taking place and that this repair *is part of the analysis and optimisation of the network*. This feature distinguishes

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repairable flow networks from *static flow networks* [1–7] and *stochastic flow networks* [8–13], for which no repair of failed components is ever considered.

A flow network with a single source and a sink can be presented as a directed graph $G = (V, E)$ consisting of a set of nodes V and a set of edges E (components) [14]. Each edge/component (i, j) is characterised by a flow capacity $c(i, j)$. For all nodes in the flow network different from the source and the sink, the flow conservation principle is fulfilled. The sum of the flows going into a node is equal to the sum of the flows going out of the node.

Along the edges, the capacity constraint is fulfilled. In other words, the flow $f(i, j) \geq 0$ along an edge (i, j) cannot exceed the flow capacity $c(i, j) \geq 0$ of the edge $(f(i, j) \leq c(i, j))$.

If both the flow conservation principle and the capacity constraint are fulfilled, *feasible edge flows* are present. The problem most often considered for static flow networks is finding feasible edge flows in the network which maximise the output flow [1–7].

Stochastic flow networks, where, on demand, the flow capacities of the edges are treated as random variables, have also been considered [8–13]. However, no renewal cycle, consisting of operation followed by a downtime for repair, has ever been considered for stochastic flow networks.

The problem of interest for stochastic networks was the probability that, on demand, the output flow from the network will be equal to or greater than a specified level. Maximising the flow in stochastic flow networks has been accomplished with methods involving minimal cut sets or minimal paths [8–13]. Such an approach has, for example, been adopted by Jane et al. [11], where stochastic flow networks with multistate components have been considered. Reliability evaluation of communication networks based on flow graphs has been treated by Aggarval et al. [13]. Again, the method was based on finding, one by one, all available forward paths from the source to the sink.

Although, for small-size networks, an approach based on minimal paths or minimal cut sets is acceptable, with increasing size of the network, the number of minimal paths and cut sets increases exponentially, and this approach is no longer feasible. In the present treatment, approaches based on determining cut sets or possible paths from the source to the sink have been abandoned, because of their incapability of handling large networks. With increasing size of the network, the number of cut sets and possible paths from the source to the sink increases exponentially, and even the storage of the identified cut sets or paths becomes impossible.

Analysis and optimisation of repairable flow networks is a new area of research initiated in [15], where it was demonstrated for the first time that *maximising the flow in a static flow network (where no edge failures occur) does not necessarily guarantee that the flow in the corresponding repairable network will be maximised.*

During a specified time interval of operation for the repairable network (e.g., during the lifecycle of the network), the components fail; their flow capacity is reduced to zero during a specified downtime for repair, after which the components are returned to operation.

Basic concepts, performance characteristics, and algorithms for analysis and optimisation of repairable flow networks have been proposed in [16]. One of the important performance measures of repairable flow networks is the ratio

$$\psi = \overline{Q}_r / Q_0 \tag{1}$$

of the maximum expected output flow $\overline{Q}_r$ from the repairable flow network in the presence of failures, during a specified time interval, to the maximum output flow Q_0 that could be obtained in the absence of failures. For production networks, this performance measure is also known as *average production availability*. It is commonly specified in contracts, and it is a key parameter for evaluating the performance of production systems. It is used for comparing alternative solutions and for informed selection among competing design solutions based on different network topologies.

Production availability is adversely affected by component failures. This is why there exists an urgent demand for solutions determining the network topology characterised by the largest production availability. Past experience and observation have indicated that the topology of repairable flow networks has a significant impact on their performance. Two networks built with identical type and number of components can have very different production availabilities, because of slight differences in their topology. To achieve a maximum increase of the quantity of the transmitted flow during a particular operational period, redundancies need to be placed in an optimal way.

To determine the value of the production availability, the variation of the total output flow Q_r in the presence of component failures needs to be determined. The quantity Q_r is a function of the number of failures and the time-to-failure and the time-to-repair distributions of the components. To reveal this variation, a large number of failure histories during the period of operation of the network must be simulated.

Furthermore, every repairable flow network is also associated with a specific capital cost for building it. A significant part of this cost is the sum of the costs of its components. A very important objective here is to achieve a desired production availability, at a minimal cost for building the network.

Currently, no algorithms are known to be capable of achieving this objective. Existing software tools, handling flows in networks, operate on a fixed network topology. They do not perform a repeated modification of the network topology and calculation of the production availability in order to determine the right topology, combining a minimum cost for building the network and a maximum production availability.

In the complex repairable flow networks used today, there is a large number of possibilities for selecting components with different reliabilities and costs, design configurations, cross-bridges, and redundancies (e.g., active redundancy, standby redundancy, k-out-of-n redundancy, etc.).