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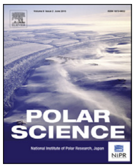

# Comparison and verification of enthalpy schemes for polythermal glaciers and ice sheets with a one-dimensional model

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## Abstract

The enthalpy method for the thermodynamics of polythermal glaciers and ice sheets is tested and verified by a one-dimensional problem (parallel-sided slab). The enthalpy method alone does not include explicitly the transition conditions at the cold–temperate transition surface (CTS) that separates the upper cold from the lower temperate layer. However, these conditions are important for correctly determining the position of the CTS. For the numerical solution of the polythermal slab problem, we consider a two-layer front-tracking scheme as well as three different one-layer schemes (conventional one-layer scheme, one-layer melting CTS scheme, one-layer freezing CTS scheme). Computed steady-state temperature and water-content profiles are verified with exact solutions, and transient solutions computed by the one-layer schemes are compared with those of the two-layer scheme, considered to be a reliable reference. While the conventional one-layer scheme (that does not include the transition conditions at the CTS) can produce correct solutions for melting conditions at the CTS, it is more reliable to enforce the transition conditions explicitly. For freezing conditions, it is imperative to enforce them because the conventional one-layer scheme cannot handle the associated discontinuities. The suggested numerical schemes are suitable for implementation in three-dimensional glacier and ice-sheet models.

*Keywords:* Glacier; Ice sheet; Polythermal ice; Modeling; Enthalpy method

## 1. Introduction

The decrease of the ice viscosity with increasing content of liquid water in temperate ice was first confirmed and measured by Duval (1977). It is therefore desirable to simulate the water content in glaciers and ice sheets realistically, especially if the temperate ice occurs in a basal layer where shear deformation is largest. Mathematical models of polythermal ice masses were introduced and further developed by Fowler and Larson (1978), Hutter (1982), Fowler (1984) and Hutter (1993). We distinguish essentially two types of polythermal glaciers, Canadian-type polythermal glaciers, which are cold in most of the ice mass except for a temperate basal layer

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in the ablation zone, and Scandinavian-type glaciers, which are temperate in most parts except for a cold surface layer in the ablation zone (Fig. 1) (Blatter and Hutter, 1991).

This work attempts to verify thermodynamic schemes used in shallow ice sheet models. Therefore, we do not investigate processes which are not usually included in ice sheet models, such as possible diffusion of water in temperate ice (Hutter, 1993) and pre-melting in ice at sub-freezing temperatures (Dash et al., 2006). For verification of numerical solutions with exact solutions, we neglect the pressure dependence of the melting point and the temperature dependence of the heat conductivity and specific heat capacity. Following Aschwanden and Blatter (2005), “ice is treated as temperate if a change in heat content leads to a change in liquid water content alone, and is considered cold if a change in heat content leads to a temperature change alone.” This implies that temperate ice is at the melting point and the temperatures in cold ice are below the melting point.

Polythermal schemes that solve the field equations for the cold and temperate layers separately were implemented for both types of polythermal glaciers, in one dimension for the Scandinavian-type Storglaciären, Sweden (Pettersson et al., 2007), in two dimensions for the Canadian-type Laika Glacier, Canada (Blatter and Hutter, 1991) and for three-dimensional ice sheets, which are Canadian-type polythermal (Greve, 1997). With the assumption that water mostly accumulates along the trajectories of ice particles in the temperate layer, Aschwanden and Blatter (2005) used a trajectory model to determine the position of the cold-temperate transition surface (CTS) and the water content in the temperate part of Storglaciären. Aschwanden et al. (2012) suggested an enthalpy scheme with the idea that, with enthalpy, only one thermodynamic field variable must be computed, and the temperature and water content result from the enthalpy as diagnostic fields. The domains of cold and temperate ice are discriminated by the contour of the enthalpy of ice with no liquid water content at the melting point.

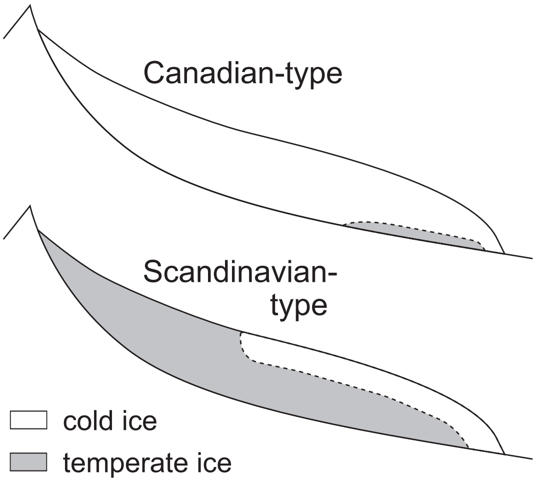

Fig. 1. Schematic cross sections of Canadian- and Scandinavian-type polythermal glaciers (adapted from Aschwanden et al., 2012).

A crucial point in polythermal enthalpy schemes is their treatment of the Stefan-type energy- and mass-flux matching conditions at the CTS, which are important for determining its position (Greve, 1997). These transition conditions are not included explicitly in the formulation of the enthalpy scheme according to Aschwanden et al. (2012).

Two different cases must be distinguished. Melting conditions occur if cold ice flows across the CTS into the temperate region. At the CTS, the particles consist of ice at melting temperature without liquid water, and, after the transition, start to accumulate water due to strain heating. Thus, the boundary condition on the temperate side of the CTS is zero water content. To match the vanishing latent heat flux, the diffusive heat flux and corresponding enthalpy gradient on the cold side must also vanish.

The situation is different for freezing conditions at the CTS, where the ice flows from the temperate region into the cold region and the liquid water content of the temperate ice freezes at the CTS. The advective latent heat flux on the temperate side then changes into a diffusive heat flux on the cold side. Thus, a drop of a non-vanishing water content to zero results in a non-vanishing temperature (enthalpy) gradient in the cold layer at the CTS.

This work attempts to verify and test modified enthalpy methods, and in particular to test how the modified schemes handle the internal boundary between cold and temperate ice. For the verification, we use an exact solution which is available for steady states in a parallel-sided slab, which reduces the problem to one dimension (Greve, 1997; Greve and Blatter, 2009). In Section 2, we review the main concepts of the enthalpy method, and in Section 3, we formulate the enthalpy method for the special case of the parallel-sided slab. Section 4 deals with different one- and two-layer methods to solve this problem, the two-layer front-tracking scheme being used to provide reference solutions against which the performance of the simpler one-layer methods can be tested. Concrete numerical experiments are defined in Section 5,