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# Adaptive registration of magnetic resonance images based on a viscous fluid model

Herng-Hua Chang*, Chih-Yuan Tsai

Computational Biomedical Engineering Laboratory (CBEL), Department of Engineering Science and Ocean Engineering, National Taiwan University, Daan, 10617 Taipei, Taiwan

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ABSTRACT

This paper develops a new viscous fluid registration algorithm that makes use of a closed incompressible viscous fluid model associated with mutual information. In our approach, we treat the image pixels as the fluid elements of a viscous fluid governed by the nonlinear Navier–Stokes partial differential equation (PDE) that varies in both temporal and spatial domains. We replace the pressure term with an image-based body force to guide the transformation that is weighted by the mutual information between the template and reference images. A computationally efficient algorithm with staggered grids is introduced to obtain stable solutions of this modified PDE for transformation. The registration process of updating the body force, the velocity and deformation fields is repeated until the mutual information reaches a prescribed threshold. We have evaluated this new algorithm in a number of synthetic and medical images. As consistent with the theory of the viscous fluid model, we found that our method faithfully transformed the template images into the reference images based on the intensity flow. Experimental results indicated that the proposed scheme achieved stable registrations and accurate transformations, which is of potential in large-scale medical image deformation applications.



## 1. Introduction

Image registration is one of the fundamental and essential tasks within image processing and analysis. It is the process of determining the correspondence between structures in two images, which are respectively called the template (source) image and the reference (target) image. The challenge of registration is to find an optimal geometric transformation between corresponding image data. In other words, given a reference image and a template image, the goal is to find a suitable transformation such that the transformed template image becomes similar or even identical to the reference image. Recently, there is an increasing need for the registration of medical images, for example, magnetic resonance (MR) images, in many research and clinical applications such as diagnosis, therapy and surgery planning, and tracking of physical deformations (e.g., tumor growth, brain atrophy) [1–4]. In essence, the intention is to match two or more images that depict the same anatomical structures at different times or from different subjects or from different imaging modalities.

A number of registration methods based on different physical theories and mathematical formulas have been proposed [3,5–12]. Transformations of maintaining distances between all pixels in images are referred to as rigid-body transformations that are based on coordinate changing by translation and

* *Corresponding author at*: 1 Sec., 4 Roosevelt Road, Daan, 10617 Taipei, Taiwan. Tel.: +886 2 3366 5745; fax: +886 2 2392 9885.
E-mail addresses: herbertchang@ntu.edu.tw, emwave@ucla.edu (H.-H. Chang).
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rotation. In contrast, non-rigid transformations map straight lines into curves in such a way as to determine the transformation function of an object between two images. Non-rigid methods are often used to handle large scale and complicated deformations such as the registration of medical images [13]. In general, non-rigid transformation models can be broadly divided into two major categories: physical based models and function representations [13,14]. In particular, the physical models are derived from the theory of continuum mechanics and can be divided into three main subcategories: optical flow, linear elasticity, and fluid flow [13].

Optical flow methods have been used to find small scale deformations in temporal sequences of images. The basic assumption of optical flows is based on the principle of intensity conservation between image frames. For image registration, the motion equation of optical flows is numerically approximated to achieve stable transformations for the guidance of the displacement, e.g., the demons algorithm [15]. Furthermore, Gaussian convolution is introduced to smooth the displacement map to reduce the influences of noise. A disadvantage of this model is that there is no constraint on the displacement field and the template image is resampled at each iteration [13,16]. Subsequently, Wang et al. [17] accelerated the demons algorithm [15] by introducing an active force along with an adaptive force strength adjustment during the iterative process. Noticeable improvements on the computation speed and a higher tolerance of large organ deformations were achieved using this method.

The basic idea underlining linear elasticity is based on (a) stress, which is the contact force per unit area acting on orthogonal planes, and (b) strain, which is a measure of the amount of deformation. Considering a linearly elastic material in force equilibrium and assuming that the stress components vary linearly across an infinitesimal element, it can be shown that the stress tensor is symmetric and the number of independent stress components can be reduced to six [18]. For a homogeneous isotropic material the stress–strain relationship can be simplified to the Piola–Kirchhoff form, from which we obtain the Navier–Cauchy linear elastic partial differential equation (PDE) as follows:

$$\mu\nabla^2\vec{d} + (\mu+\lambda)\nabla(\nabla\cdot\vec{d}) + \vec{f} = 0, \tag{1}$$

where $\mu$ and $\lambda$ are the Lamé constants, $\vec{d}$ represents the displacement, $\nabla^2 = \nabla\cdot\nabla$, $\nabla$ is the gradient operator, and $\vec{f}$ denotes the body force per unit volume, which drives registration. Eq. (1) is the governing transformation equation for linear elastic registration that is only accurate for small deformations [13,18]. For larger deformations, elastic displacements do not reach the desired transformation because the deformation is a compromise between internal and external forces [19].

Fluid registration is of particular importance in medical image analysis, as it can be used to localize regions of anatomical change in longitudinal studies and accommodate large deformations while maintaining a diffeomorphism [20–22]. A diffeomorphism is a globally one-to-one smoothing and continuous transformation with differential derivatives that are invertible [23]. Generally speaking, viscous fluid registration methods require the solutions to large sets of PDEs and continuum mechanics provides the theoretical foundation for fluid flow registration. Fluid flow models are based on physical properties of fluids that follow Newtonian mechanics and must satisfy physical laws such as the conservation of linear momentum that leads to the equation of motion. For a Newtonian fluid the viscous stress tensor is linearly related to the rate of the deformation tensor. By assuming Stokes flow (low Reynolds number) and only a small spatial variation in the hydrostatic pressure, we obtain the simplified Navier–Stokes equation for a viscous fluid as [13,18,24]

$$\mu_f\nabla^2\vec{V} + (\mu_f+\lambda_f)\nabla(\nabla\cdot\vec{\nabla}) + \vec{f} = 0, \tag{2}$$

where $\mu_f$ and $\lambda_f$ are the viscosity coefficients of the fluid, $\vec{V}$ represents the velocity vector, the term $\mu_f\nabla^2\vec{V}$ indicates constant volume or incompressible viscous flow and the $(\mu_f+\lambda_f)\nabla(\nabla\cdot\vec{\nabla})$ term controls expansion or contraction of the fluid. Note that the Navier–Stokes PDE (2) is equivalent to the Navier–Cauchy linear elastic PDE (1) with the variable velocity $\vec{V}$ replaced by displacement $\vec{d}$.

This viscous fluid model of the dynamical PDE (2) was proposed by Christensen et al. [24,25] to allow large magnitude deformations. In their approach, image pixels are regarded as viscous fluid particles that are governed by the PDEs, which constrain the movement during the registration process. To numerically solve the PDEs, the authors used conventional finite difference techniques associated with the successive over-relaxation (SOR) scheme. A regridding procedure is required when the nonlinear transformations evaluated on a finite lattice become singular. Subsequently, Freeborough and Fox [26] adopted a full multi-grid (FMG) framework along with the SOR for the coarse solution to match a set of Alzheimer disease (AD) images. Crum et al. [27] used semi-coarsening and exploited the inherent multi-resolution nature of FMG to implement a multi-scale approach. Xu and Dony [2] used the least mean square inverse filter to solve the PDEs of the viscous fluid model [24]. D'Agostino et al. [28] adopted a multimodal viscous fluid model registration algorithm based on maximization of mutual information. Rong et al. [29] presented a fast registration method by combining the traditional viscous fluid model with B-spline and fast Fourier transformation to accelerate the computation.

Motivated by the success of the existing viscous fluid models and mutual information frameworks [1,4,10,26–30], we propose a variational registration algorithm based on the theory of fundamental fluid mechanics. The ambition of this paper is in an attempt to develop an adaptive image registration algorithm that makes use of a closed incompressible viscous fluid model associated with mutual information. Unlike the existing fluid registration methods, the governing equation of the proposed framework is based on the temporally and spatially variant Navier–Stokes PDE. For registration, the pressure term is replaced with a body force that computes the intensity differences between the template image and the reference image to drive the flow. To adaptively control the evolution and to accelerate the process, the body force is associated with mutual information, which is an entropy function of the similarity between the two images. A computationally efficient technique that decomposes the governing PDE into two components in 2-D is proposed to compute the