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Geodesy and Geodynamics

Analysis of the impact on the gravity field determination from the data with the ununiform noise distribution using block-diagonal least squares method

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ABSTRACT

The block-diagonal least squares method, which theoretically has specific requirements for the observation data and the spatial distribution of its precision, plays an important role in ultra-high degree gravity field determination. On the basis of block-diagonal least squares method, three data processing strategies are employed to determine the gravity field models using three kinds of simulated global grid data with different noise spatial distribution in this paper. The numerical results show that when we employed the weight matrix corresponding to the noise of the observation data, the model computed by the least squares using the full normal matrix has much higher precision than the one estimated only using the block part of the normal matrix. The model computed by the block-diagonal least squares method without the weight matrix has slightly lower precision than the model computed using the rigorous least squares with the weight matrix. The result offers valuable reference to the using of block-diagonal least squares method in ultra-high gravity model determination.

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1. Introduction

The Earth's gravity field is the basic physical field of the Earth, which has an important role in geodesy and geophysics. Surface grid data such as the surface grid gravity anomaly data are crucial to the determination of the Earth's gravity filed. Numerical quadrature technique and least squares method are main methods to compute the gravity field using surface grid data. Different from numerical quadrature technique, least squares method can evaluate the precision of the computed model coefficients, and the least squares method is widely used in gravity field determination [1–6]. However in ultra-high degree gravity field determination, the number of parameters and the dimension of the normal matrix are huge. Under current computation conditions it might be impossible to use the least squares method directly. Fortunately, spherical harmonic functions are orthogonal. When the observation data and precision distribution of the data satisfy the specific requirements and one arranges the parameters in a particular sequence, the normal matrix in the least squares method is block-diagonal matrix. Using block-diagonal least squares method to compute the model coefficients, may greatly reduce the calculation quantity. This issue has been studied in some papers [4,7,8]. Colombo [7] firstly described the structure of the normal matrix. Pavlis [8] compared the block-diagonal least squares method with the numerical quadrature method in gravity filed determination. Li Xinxing [4] focused on the detailed application of the block-diagonal method in gravity field modeling. Lemoine [9], Pavlis [1] and Förste [10–12] all used the block-diagonal least squares method in gravity field determination.

The necessary condition of the block-diagonal least squares method for gravity field modeling is that the weight of surface grid data must be independent of longitude which means the weight at the same latitude line must be equal. But the actual observation data can't satisfy this condition, so the normal matrix is not block-diagonal, which brings difficulty to the using of block-diagonal method. In ultra-high gravity model determination, the computation amount is so huge that one has to use the block-diagonal least squares method to simplify the computation amount. Therefore, we intend to research on the data processing strategy for the situations when data don't satisfy the requirements and give some valuable advice to the using of block-diagonal least squares method. This paper firstly studies the forms of the normal matrix that formed when using least squares method, and then analyzes the results of three data processing strategies in gravity field determination through numerical simulation.

2. Basic principle

The relation between surface grid gravity anomaly observations and gravity model coefficients is [2]:

$$\Delta g_{ij}^{c}=\frac{GM}{r_{ij}^{2}}\sum_{n=2}^{\infty}(n-1)\left(\frac{R}{r_{ij}}\right)^{n}\sum_{m=0}^{n}(\overline{C}_{nm}\cos m\lambda_j+\overline{S}_{nm}\sin m\lambda_j)\overline{P}_{nm}(\cos\theta_i) \tag{1}$$

where (*i,j*) represent the grid on the sphere or ellipsoid surface at the *i*-th row and *j*-th column. $\overline{P}_{nm}(\cos\theta)$[13] is the normalized associated legendre function. $\overline{C}_{nm},\overline{S}_{nm}$ are gravity model coefficients. Δg_{ij}^{c} is the gravity anomaly in which the ellipsoid correction, atmosphere correction, second-order normal gravity gradient correction and analytical downward continuation correction are taken, which satisfies the basic equation of boundary value problem. The derivation of this formula can be found in the reference [2].

Based on equation (1), one can form the observation equation and the normal equation using the least squares method [14]. Then the model coefficients can be computed based on the normal equation. The observation equation is:

$$\boldsymbol{v}=\boldsymbol{Ax}-\boldsymbol{L} \tag{2}$$

Every parameter in equation (1) has a corresponding coefficient function, and the element of the design matrix **A** in equation 2 is the value of the coefficient function $\frac{GM}{r_{ij}^{2}}(n-1)\left(\frac{a}{r}\right)^{n}\begin{pmatrix}\cos\\ \sin\end{pmatrix}(m\lambda_j)\overline{P}_{nm}(\cos\theta_i)$ corresponding to a certain parameter taking at the (*i,j*) grid on the surface. The elements of matrix **A** in the same column are the values that a same coefficient function $\frac{GM}{r_{ij}^{2}}(n-1)\left(\frac{a}{r}\right)^{n}\begin{pmatrix}\cos\\ \sin\end{pmatrix}(m\lambda_j)\overline{P}_{nm}(\cos\theta_i)$ takes at all the surface grids. The element of the normal matrix **N** is the inner product of two columns in the design matrix **A**, which means that the element of the normal matrix **N** is the inner product on the surface of two coefficient functions corresponding to two parameters. If an element of the normal matrix **N** is the inner product on the surface of two coefficient functions corresponding to the parameter C_{nm}^{α} and C_{rs}^{β}, then the element can be written as:

$$[\mathbf{N}]_{C_{nm}^{\alpha}C_{rs}^{\beta}}=GM^{2}(n-1)(r-1)\sum_{i=0}^{N-1}\overline{P}_{nm}^{i}\overline{P}_{rs}^{i}\sum_{j=0}^{2N-1}\frac{1}{r_{ij}^{4}}\left(\frac{a}{r_{ij}}\right)^{n+r}\begin{Bmatrix}C_m^j\\ S_m^j\end{Bmatrix}\begin{Bmatrix}C_s^j\\ S_s^j\end{Bmatrix}P_{ij} \tag{3}$$

In ultra-high degree gravity field determination, the dimension of the normal matrix is huge, so it is impossible to use the least squares method directly under current computation condition. When the observation data satisfies the specific requirements and one arranges the parameters in a particular sequence, block-diagonal least squares method can be used in the computation. At first one can divide the normal equation into several “block” equations due to the form of the normal matrix and then solve each “block” equation. The set of the solutions of the “block” equations are the solution of the normal equation. In this way the gravity field coefficients can be obtained. The block-diagonal least squares method needs much less amount of computation