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An improved algorithm for numerical calculation of seismic response spectra

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ABSTRACT

The information of seismic response spectra is key to many problems concerned with aseismic structure and is also helpful for earthquake disaster relief if it is generated in time when earthquake happens. While current numerical calculation methods suffer from poor precision, especially in frequency band near Nyquist frequency, we present a set of improved parameters for precision improvement. It is shown that precision of displacement and velocity response spectra are both further improved compared to current numerical algorithms. A uniform fitting formula is given for computing these parameters for damping ratio range of 0.01–0.9, quite convenient for practical application.

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1. Introduction

Seismic response spectra contain not only characteristic of seismic spectra but also information of maximum response of simple structure for earthquake movement. These information are key to many problems concerned with aseismic structure and is also helpful for earthquake disaster relief if they are generated in time when the earthquake occurs. Seismic response spectra can be calculated using accelerograph record in frequency or time domain. While the former can be done only when the whole seismic event record is available, the latter can be real-time in recursive way. Earthquake warning and earthquake intensity rapid report system is emphasized nowadays to minimize earthquake disaster where a large amount of earthquake intensity meters based on MEMS accelerator are usually set up due to their low cost and easy installation. The amount of data is then so large that it is difficult for computers in center to process all of them to get real-time result. The earthquake intensity meter is required to produce necessary outcomes such as seismic

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response spectra in real time itself, which is possible only using time domain methods. There are several methods presented nowadays such as center difference method, Newmark method [1], Z-transformation method [2] and Duhamel step-by-step integration method [3–5]. As stated by Ma Qiang et al. [6] and Jing Xing et al. [7], these methods still suffer from poor precision and they present an improved method (see next section in detail). We find that further improvement is possible after examining these methods and present another set of improved parameters to get better precision than current main numerical algorithms.

2. Precision of current main numerical algorithms

Seismic response spectra are defined as maximum response of single degree of freedom (SDOF) system for earthquake acceleration input:

$$\ddot{x} + 2h\omega_0\dot{x} + \omega_0^2 = -a \quad (1)$$

where h is damping ratio, ω_0 is natural angular frequency, a is earthquake acceleration and x is response displacement for earthquake acceleration input.

The transfer function of response displacement, velocity and acceleration relative to earthquake acceleration are respectively given by equations (2)–(4):

$$H_p(\omega, h, \omega_0) = \frac{-1}{-\omega^2 + 2ih\omega_0\omega + \omega_0^2} \quad (2)$$

$$H_v(\omega, h, \omega_0) = \frac{-i\omega}{-\omega^2 + 2ih\omega_0\omega + \omega_0^2} \quad (3)$$

$$H_a(\omega, h, \omega_0) = \frac{\omega^2}{-\omega^2 + 2ih\omega_0\omega + \omega_0^2} \quad (4)$$

The response spectra of displacement, velocity and acceleration can be calculated through multiplying spectra of earthquake acceleration by these transfer functions in frequency domain when the spectra of earthquake acceleration are known. This is precise but can be done only when the whole record of earthquake event is available and is not real-time. There are many time-domain algorithms such as center difference method, Newmark method [1], Z-transform method [2] and Duhamel step-by-step integration method [3–5], Jin Xing et al. [7] has compared these methods in detail and finally recommended the following algorithm (subscript j denotes No. j sample or computed value) and made a further study on it in another reference [8]:

$$x_j = b_1x_{j-1} + b_2x_{j-2} - S_0(\Delta t)^2\left[\delta a_j + (1-2\delta)a_{j-1} + \delta a_{j-2}\right] \quad (5)$$

$$\dot{x}_j = b_1\dot{x}_{j-1} + b_2\dot{x}_{j-2} - S_0\Delta t\left(0.5a_j - 0.5a_{j-2}\right) \quad (6)$$

$$\ddot{x}_j = b_1\ddot{x}_{j-1} + b_2\ddot{x}_{j-2} - S_0\left(a_j - 2a_{j-1} + a_{j-2}\right) \quad (7)$$

where,

$$b_1 = 2e^{-h\omega_0\Delta t}\cos(\omega_d\Delta t) \quad (8)$$

$$b_2 = -e^{-2h\omega_0\Delta t} \quad (9)$$

$$S_0 = (1 - b_1 - b_2)\big/(\omega_0\Delta t)^2 \quad (10)$$

$$\omega_d = \omega_0\sqrt{1-h^2} \quad (11)$$

and Δt is time interval of sampling.

δ in equation (5) is given by the following equation as Ma Qiang et al. [6]:

$$\delta = d_1 + d_2(\Delta t/T_0) + d_3(\Delta t/T_0)^2 + d_4(\Delta t/T_0)^3 \quad (12)$$

where T_0 is natural period. When the range of damping ratio is 0.01–0.1, the coefficients d_1–d_4 are fitted by

$$d_1 = 0.09108 \quad (13)$$

$$d_2 = 0.01945 - 0.04679h \quad (14)$$

$$d_3 = -0.00989 + 0.38022h \quad (15)$$

$$d_4 = 0.50617 - 0.97476h \quad (16)$$

When the values of damping ratio are 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9, the coefficients d_1–d_4 are listed in a table in reference [6].

Because the precision of equation (6) is not so good, Jin Xing et al. [9] presented the following improved equation:

$$\dot{x}_{j-1/2} = b_1\dot{x}_{j-3/2} + b_2\dot{x}_{j-5/2} - S_0\Delta t\left[\delta a_j + (1-3\delta)a_{j-1} - (1-3\delta)a_{j-2} - \delta a_{j-3}\right] \quad (17)$$

where δ is the same as in equation (12).

We call the method presented by Jing Xing et al. in references [7,9] as Jing Xing method for convenience in this paper. The relative error of response amplitude spectra of displacement, velocity and acceleration are calculated and shown in Fig. 1 using the above methods when sample rate is 100 Hz, damping ratio 0.05, natural period 0.3 s and frequency band 0.01–50 Hz. It is clear that the errors of acceleration response spectra are less than 5% and the precision of Duhamel method and Jing Xing method are relatively better. But the errors of velocity and displacement response spectra increase with frequency and become poor in high frequency band except Jing Xing method. Among all of the above methods, the precision of Jin Xing method is actually improved but is still poorer than 5% in 40–50 Hz for velocity and displacement response spectra.

3. Improved algorithm

The error of velocity response spectra of Jin Xing method is poorer than that of displacement response spectra because they use the same value of δ to optimize displacement recursive equation (equation (5)) and velocity recursive equation (equation (17)) which may not perform well in response velocity calculation. We use different δ (δ_p and δ_v) for displacement and velocity recursive equation as:

$$x_j = b_1x_{j-1} + b_2x_{j-2} - S_0(\Delta t)^2\left[\delta_p a_j + (1-2\delta_p)a_{j-1} + \delta_p a_{j-2}\right] \quad (18)$$

$$\dot{x}_{j-1/2} = b_1\dot{x}_{j-3/2} + b_2\dot{x}_{j-5/2} - S_0\Delta t\left[\delta_v a_j + (1-3\delta_v)a_{j-1} - (1-3\delta_v)a_{j-2} - \delta_v a_{j-3}\right] \quad (19)$$