Geodesy and Geodynamics 2013,4(4):9-16

http://www.jgg09.com
Doi:10.3724/SP.J.1246.2013.04009

Theoretical simulation of the effect of deformation on local gravity in a density gradient zone

Zhu Liangyu[1,2], Wang Qingliang[2] and Zhu Yiqing[2]
[1] *Institute of Geology, China Earthquake Administration, Beijing* 100029, *China*
[2] *Second Crust Monitoring and Center, China Earthquake Administration, Xi'an* 710054, *China*

Abstract: We modeled the effect of the deformation of a Density Gradient Zone (DGZ) on a local gravity field using a cubical model and introduced a new method to simulate a complex DGZ (CDGZ). Then, we analyzed the features of the model for the influence of the deformation of the DGZ on the local gravity field. We concluded that land-based gravity is not sensitive to the thickness of the DGZ and that the magnitude of the contribution of the DGZ is one order less than that of the volume strain with the same displacement.
Key words: Density Gradient Zone (DGZ); density interface; cubical model; vertical deformation; gravity anomaly

1 Introduction

Following the pioneering work of Walsh[1], who derived the integral expression between local crust deformation and gravity changes on a fixed point in space by assuming isotropic density and infinitesimal deformation, numerous researchers have attempted to extend the theory and apply it to explanations of the changes in gravity and deformations that occur before earthquakes. Reilly and Hunt[2,3] investigated Walsh's formula and noted a mistake, but their modified expression still ignored the effect due to the deformation of the lower surface surrounding the integral volume. To study the mechanism of the changes in gravity that occurred prior to two earthquakes at Haicheng City in 1975 (Ms = 7.5) and Tangshan City in 1976 (Ms = 7.8), Chen[4] derived a general expression about the changes in gravity caused by the deformation of a continuous medium with a cavity, including the influence of ground vertical deformation. More recently, Shen[5] extended the integral expression to large-scale crustal movement to allow the simulation of the coupling of regional crust deformation and changes in gravity. However, a practical formula was not available because of the complex integral region and integrable functions.

In the present study, we modeled the effect of deformation due to a Density Gradient Zone (DGZ) on local gravity using a cubical model and introduced a new method to simulate a Complex Density Gradient Zone (CDGZ). Then, we analyzed the features of the model for the influence of the deformation of the DGZ on the local gravity field. We concluded that land-based gravity is not sensitive to the thickness of the DGZ and that the effect of the DGZ is one order less than that of the volume strain with the same displacement.

2 Changes in gravity due to the deformation of a DGZ

According to the general formula, the changes in gravity due to the deformation in a continuous medium with

Received: 2013-02-05; Accepted: 2013-04-06
Corresponding author: Zhu Liangyu, E-mail: hehaizhuliangyu@163.com.
This research was supported by the Special Earthquake Research Project of China Earthquake Administration (201208009) and the National Natural Science Foundation of China (41274083).

a cavity on an observed point fixed in space $P(x_0, y_0, z_0)$ can be described as follows[4]:

$$\boldsymbol{\delta g} = -G\iiint_V \frac{z\nabla\cdot(\rho \boldsymbol{u})}{R^3}dV + G\iint_S \frac{z\rho \boldsymbol{u}\cdot \boldsymbol{n}}{R^3}dS - 2\pi G\left(\frac{4}{3}\rho_E - \rho\right)\boldsymbol{h} \tag{1}$$

where $\boldsymbol{\delta g}$ is the change in gravity, G is the gravitational constant, R is the distance between the observed point $P(x_0, y_0, z_0)$ and the deformed point in volume V, z is the depth of the deformed point, ρ is the density of the medium, $\boldsymbol{u}$ is the displacement of the deformed point, S represents the outside and inside surfaces of the deformed volume V, excluding the Earth's surface, $\boldsymbol{n}$ is the outside normal of S, ρ_E is the average density of the Earth (5.517 g/cm^3), $\boldsymbol{h}$ is the vertical deformation, and ∇ is the divergence operator.

The first term of formula (1) can be regarded as the effect of inner deformation for a deformed volume V on the local gravity field[4, 6, 7]. In general, if the density of the deformed volume is constant in an isotropic medium, then the first term of formula (1) degenerates into the influence of volume strain. However, the property of the crust varies with space significantly, so the density of the deformed volume cannot be assumed as constant with long-term tectonic activity. This procedure can be described as follows[8]:

$$\nabla(\rho\boldsymbol{u}) = \rho\nabla\cdot\boldsymbol{u} + \boldsymbol{u}\cdot \mathrm{grad}\rho = \rho\theta\mathbf{I} + \boldsymbol{u}\cdot\mathrm{grad}\rho \tag{2}$$

where $\theta\mathbf{I} = \nabla\boldsymbol{u}$ is the volume strain, and gradρ is the vector of the density gradient.

Substituting equation (2) into (1) and defining $\boldsymbol{\delta g_\theta}$ and $\boldsymbol{\delta g_2}$, we obtain:

$$\begin{aligned}\boldsymbol{\delta g_\theta} &= -G\iiint_V \frac{z\rho\theta\mathbf{I}}{R^3}dV \\ &= -G\iiint_V \frac{z\rho\boldsymbol{\varepsilon}_x}{R^3}dV - G\iiint_V \frac{z\rho\boldsymbol{\varepsilon}_y}{R^3}dV - G\iiint_V \frac{z\rho\boldsymbol{\varepsilon}_z}{R^3}dV\end{aligned} \tag{3}$$

$$\boldsymbol{\delta g_2} = -G\iiint_V \frac{z\boldsymbol{u}\cdot\mathrm{grad}\rho}{R^3}dV \tag{4}$$

where $\boldsymbol{\varepsilon}_x$, $\boldsymbol{\varepsilon}_y$ and $\boldsymbol{\varepsilon}_z$ are line strains for the x-, y-, and z-directions, respectively. Therefore, it is important to quantitatively estimate the change in gravity caused by the DGZ and volume strain and to analyze their features.

2.1 Changes in gravity due to the vertical displacement of the DGZ

As shown in figure 1, the cubical model was applied to simulate the changes in gravity due to the vertical displacement of the DGZ. The parameters of the cubical model were designed as follows: the density of the blue region is ρ_{1z}, the density of the yellow region is ρ_{2z}, the red region is the DGZ with a range from Z_1 to Z_2, and $\boldsymbol{u}\cdot$ gradρ is defined as follows:

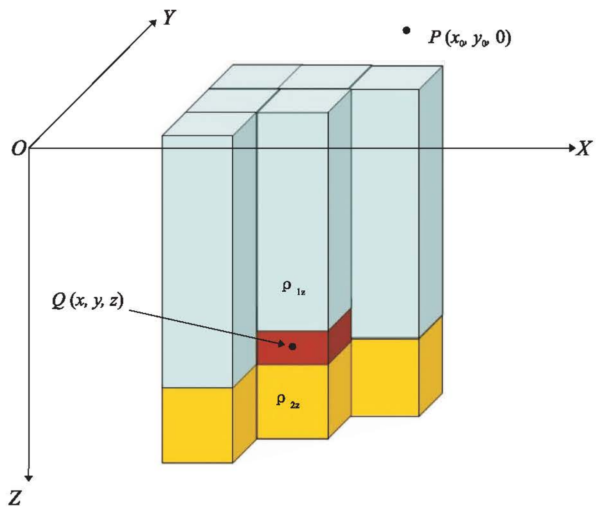

Figure 1 Model of the vertical DGZ

$$\boldsymbol{u}\cdot\mathrm{grad}\rho = \begin{cases} 0 & 0 \leqslant z \leqslant Z_1 \\ \dfrac{U_z\rho_{\Delta z}}{Z_2 - Z_1} & Z_1 < z < Z_2 \\ 0 & Z_2 \leqslant z < +\infty \end{cases} \tag{5}$$

where $\rho_{\Delta z}$ is $\rho_{2z} - \rho_{1z}$.

Substituting equation (5) into (4) and considering the assumption of infinitesimal deformation, we obtain:

$$\boldsymbol{\delta g_{2z}} = \begin{cases} -GU_z\rho_{\Delta z}\displaystyle\iiint_{x_1y_1Z_1}^{x_2y_2Z_2}\frac{z}{R^3(Z_2 - Z_1)}dxdydz & Z_2 \neq Z_1 \\ -GU_z\rho_{\Delta z}\displaystyle\lim_{Z_2\to Z_1}\frac{\iiint_{x_1y_1Z_1}^{x_2y_2Z_2}\frac{z}{R^3}dxdydz}{Z_2 - Z_1} & Z_2 = Z_1 = H \end{cases}$$