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Application of a simultaneous iterations reconstruction technique for a 3-D water vapor tomography system

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Abstract: The simultaneous iterations reconstruction technique (SIRT) is one of several reconstruction algorithms of the ART family. It is used widely in tomography because of its convenience in dealing with large sparse matrices. Its theoretical background and iteration model are discussed at the beginning of this paper. Then, the implementation of the SIRT to reconstruct the three-dimensional distribution of water vapor by simulation is discussed. The results show that the SIRT can function effectively in water vapor tomography, obtain rapid convergence, and be implemented more easily than inversion.

Key words: SIRT; reconstruction; water vapor; tomography; iteration

1 Introduction

The signals transmitted from Global Navigation Satellite Systems (GNSS) to receivers usually involve path delays when they cross the troposphere due to neutral atmosphere refraction. Recently, much research on the tomographic reconstruction of tropospheric water vapor using signal delays has been carried out, and the results have been used for short-term weather forecasting and disaster monitoring. Due to variations in the atmosphere, particularly the uncertainty in the spatial distribution of water vapor, the development of high-precision positioning is limited, though GNSS tomography can provide good estimates of the distribution of water vapor, which should benefit atmospheric correction in navigation and positioning.

However, water vapor tomography is affected by observational conditions. The method of solving normal equations can be generally summarized by the following three aspects.

1) In an ideal situation, there is at least one station in each grid cell observed from the horizontal plane. The inversion of a normal equation can be performed directly. However, it is difficult to satisfy this situation using the existing GNSS network.

2) The distribution of slant paths is inhomogeneous in the atmosphere because of the bad geometry of the GNSS network, which eventually leads to a cluster of slant delays in some grid cells. Then, the singular value decomposition technique [1] computes the eigenvalues of the system and identifies the null space of the solution, characterized by zero eigenvalues. The inversion of the overdetermined part can be performed directly [2].

3) The constraint equations not only provide information but also stabilize the normal equation. However, the variety of observations increases the difficulty of least-square inversion and prior weighting. The Kalman filter algorithm [3] can circumvent these limitations [4]; however, the initial value of the state vector and the variance-covariance matrix are difficult to determine.

In recent years, the algebraic reconstruction technique has been continually used in tomographic recon-

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struction. This method provides fast algorithms that lead to stable results without many iterations [5]. The simultaneous iterative reconstruction technique (SIRT) is one such method. It has been increasingly applied in ionospheric tomographic reconstruction due to its convenience of use [6,7]. The SIRT is not the same type of iteration method as the ART, which estimates a correction for each grid cell and adds this term to the preceding value. The SIRT evaluates only one correction term for each grid cell that considers all observations. The correction term is therefore independent on the order of the constant vector. In this way, the efficiency and precision are both improved. This study focuses mainly on the application of the SIRT in troposphere tomography to reduce the computation time and increase the efficiency of near-real weather forecasting and disaster monitoring.

2 The theory of water vapor tomography

It is assumed that the distribution of water vapor in each grid cell is homogeneous during a short period. When a GPS signal crosses the atmosphere, it is cut into several lines by supposed grid cells in the atmosphere; thus, the observation equation [8] can be described by

$$\sum a_{i,j,k} x_{i,j,k} = swv \tag{1}$$

The matrix form is

$$\boldsymbol{AX} = \boldsymbol{SWV} \tag{2}$$

where $\boldsymbol{A}$ is the coefficient matrix with all intercepts crossing the grid cells, $\boldsymbol{X}$ is the unknown parameter vector that represents water vapor, and $\boldsymbol{SWV}$ is a constant vector whose components represent the total water vapor along a slant path. Due to the large number of grid cells and the nonuniform distribution of observations, some grid cells will not be covered. To resolve this problem, horizontal and vertical constraint equations are introduced. The tomography model is as follows:

$$\begin{pmatrix} SWV \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} A \\ H \\ V \end{pmatrix} X + \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} \tag{3}$$

where $\boldsymbol{H}$ and $\boldsymbol{V}$ are the horizontal and vertical coefficients, respectively.

3 The simultaneous iterative reconstruction technique

The SIRT is, in some ways, an improvement of the ART. It evaluates only one correction term for each grid cell that considers all observations. The correction term is therefore independent of the order of the constant vector [9,10]. The iteration can be described by

$$x_j^{k+1} = x_j^k + \sum_{i=1}^{m} \lambda a_{ij} \frac{swv_i - \sum_{j=1}^{n} a_{ij} x_j^k}{\sum_{j=1}^{n} a_{ij}^2} \tag{4}$$

where x is the water vapor parameter, m is the row number of observation functions, n is the column number and also the number of parameters, a_{ij} indicates the factor in row i and column j of the coefficient matrix, swv represents the observation data, and λ is the relaxation parameter.

SIRT algorithms require several iterations to reach convergence. Usually, the criteria are defined to be small in number, which allows for the optimal termination of the iteration. Another way is based on the criterion $|SWV_0 - AX^k| = \min$, which is satisfied by computing SWV using equation (3). However, this criterion is difficult to realize for water vapor tomography by the pattern of convergence described above. In this study, we introduced the variance of Mean difference and root-mean square as the termination criteria. Because each procedure can obtain increasingly accurate results with the increase in the number of iterations if it is convergent, the algorithm will terminate after a small number of iterations ($\xi < 10^{-6}$).

Mean difference ($mdif$)

$$mdif = \frac{1}{m} \sum_{i=1}^{m} (swv_i^k - swv_i^0) \tag{5}$$