http://www.jgg09.com
Doi:10.3724/SP.J.1246.2013.02030

Hydrologically induced orientation variations of a tri-axial Earth's principal axes based on satellite-gravimetric and hydrological models

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Abstract: The Earth is a tri-axial body, with unequal principal inertia moments, A, B and C. The corresponding principal axes a, b and c are determined by the mass distribution of the Earth, and their orientations vary with the mass redistribution. In this study, the hydrologically induced variations are estimated on the basis of satellite gravimetric data, including those from satellite laser ranging (SLR) and gravity recovery and climate experiment (GRACE), and hydrological models from global land data assimilation system (GLDAS). The longitude variations of a and b are mainly related to the variations of the spherical harmonic coefficients $\bar{C}_{22}$ and $\bar{S}_{22}$, which have been estimated to be consisting annual variations of about 1.6 arc seconds and 1.8 arc seconds, respectively, from gravity data. This result is confirmed by land surface water storage provided by the GLDAS model. If the atmospheric and oceanic signals are removed from the spherical harmonic coefficients $\bar{C}_{21}$ and $\bar{S}_{21}$, the agreement of the orientation series for c becomes poor, possibly due to the inaccurate background models used in pre-processing of the satellite gravimetric data. Determination of the orientation variations may provide a better understanding of various phenomena in the study of the rotation of a tri-axial Earth.

Key words: SLR; GRACE; GLDAS; tri-axial axes; orientation variation

1 Introduction

Various studies have demonstrated that the Earth is a tri-axial body[1-10]. Let us denote the Earth's principal moments of inertia by A, B and C ($A < B < C$) for the corresponding principal axes of a, b and c[1], where c is the figure axis of the Earth and its orientation has been well studied[11]. If the Earth is a symmetric rotating body, then we have $A = B$. A symmetric rotating body was assumed in modeling the nutation[12] and polar motion[13] of the Earth, since the difference between A and B is small[1,12]. However, a more realistic tri-axial Earth should be used instead, for instance, in the explanation of the Earth's nutation[8] and in precise observations of the Earth's rotation parameters[9].

The orientations of the principal axes are related to mass distribution of the Earth[1,2]. For example, Liu and Chao[1] and Shen et al[5] suggested that b pointed along the diameter through (75.07°E, 104.93°W) and pointed at (0.000 076°N, 75.071 218°E), respective-

Received:2013-01-06; Accepted:2013-02-01

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This work is supported by National 973 Project of China (2013CB733305), NSFC (41174011; 41021061; 41128003; 41210006) and Open Research Fund Program of the Key Laboratory of Geospace Environment and Geodesy, Ministry of Education, China (110206).

ly. The orientation variations of the principal axes are related to the mass redistribution of the Earth[5, 6, 11, 14]. For example, the orientation variation of c is related to the variations of the spherical harmonic coefficients $\bar{C}_{21}$ and $\bar{S}_{21}$, which are caused by mass redistribution of the Earth[11]. Mass distribution is related to the Earth's gravity field while the orientations of the principal axes are only related to the second-degree coefficients[2]. In previous studies, the contribution of mass redistribution to the inertia moments[15] and the orientation of c have been well studied[11, 16]. Shen et al[5] and Chen et al[17] estimated the orientation variations of the principal axes, but the data they used were not sufficiently long and the discussion about the exciting mechanism was not comprehensive[17]. Studies of orientation variation of principal axes are significant for better understanding various phenomena of the rotation of a tri-axial Earth.

In this study, orientation variations of the principal axes induced by land hydrology that are derived from satellite laser ranging (SLR) and gravity recovery and climate experiment (GRACE) are compared with each other and then compared to the variation of the land hydrology model, the global land data assimilation system (GLDAS). In section 2, the method of formulating the relationship between the second-degree coefficients and the orientations of the principal axes is reviewed. The satellite-gravimetric data and hydrological models used in this study are described in section 3 and the results are shown and discussed in section 4. In section 5, conclusions and discussion are provided.

2 Method

The second-degree coefficients of Earth's gravity are related to the inertia moments of the Earth[2]. In the Earth-fixed system, the Earth's inertia moment tensor can be written as

$$\boldsymbol{I} = \int_{\text{Earth fixed}} \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{bmatrix} \mathrm{d}m \quad (1)$$

where x, y and z are coordinates, $\mathrm{d}m$ is the mass element. In the principal-axis reference frame[2, 4, 5, 17], the inertia moment tensor can be expressed as

$$\boldsymbol{I}' = \begin{bmatrix} A & & \\ & B & \\ & & C \end{bmatrix} = \int_{\text{Principal axis reference frame}} \begin{bmatrix} y'^2+z'^2 & & \\ & x'^2+z'^2 & \\ & & x'^2+y'^2 \end{bmatrix} \mathrm{d}m \quad (2)$$

where x', y' and z' are coordinates, and A, B and C are, respectively, the minimum, medium and maximum principal inertia moments. I and I' are actually two different expressions of the same tensor in different reference frames. Transformation of coordinates from one reference frame to the other can be done as follows.

The second-degree gravitational potential can be expressed as[2]

$$V_2(P) = \frac{\sqrt{15}GMa^2}{2\ r^5} r^{\mathrm{T}} H r = \frac{\sqrt{15}GMa^2}{2\ r^5} \tilde{r}^{\mathrm{T}} \tilde{H} \tilde{r} \quad (3)$$

where G is the gravitational constant, M is the mass of the Earth, and r and $\tilde{r}$ are position vectors of point in the Earth-fixed system and principal-axis reference frame, respectively. In the Earth-fixed system, the tensor can be written as[2]

$$\boldsymbol{H} = \begin{bmatrix} \bar{C}_{22} - \frac{\bar{C}_{20}}{\sqrt{3}} & \bar{S}_{22} & \bar{C}_{21} \\ \bar{S}_{22} & -\bar{C}_{22} - \frac{\bar{C}_{20}}{\sqrt{3}} & \bar{S}_{21} \\ \bar{C}_{21} & \bar{S}_{21} & 2\frac{\bar{C}_{20}}{\sqrt{3}} \end{bmatrix} \quad (4)$$

where $\bar{C}_{20}$, $\bar{C}_{21}$, $\bar{S}_{21}$, $\bar{C}_{22}$ and $\bar{S}_{22}$ are normalized second-degree coefficients. In the principal-axis reference frame, the tensor $\tilde{H}$ can be written as

$$\tilde{H} = \begin{bmatrix} \bar{A}_{22} - \frac{\bar{A}_{20}}{\sqrt{3}} & 0 & 0 \\ 0 & -\bar{A}_{22} - \frac{\bar{A}_{20}}{\sqrt{3}} & 0 \\ 0 & 0 & 2\frac{\bar{A}_{20}}{\sqrt{3}} \end{bmatrix} \quad (5)$$