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Vanishing point: Scale independence in geomorphological hierarchies

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ABSTRACT

Scale linkage problems in geosciences are often associated with a hierarchy of components. Both dynamical systems perspectives and intuition suggest that processes or relationships operating at fundamentally different scales are independent with respect to influences on system dynamics. But how far apart is "fundamentally different"—that is, what is the "vanishing point" at which scales are no longer interdependent? And how do we reconcile that with the idea (again, supported by both theory and intuition) that we can work our way along scale hierarchies from microscale to planetary (and vice-versa)? Graph and network theory are employed here to address these questions. Analysis of two archetypal hierarchical networks shows low algebraic connectivity, indicating low levels of inferential synchronization. This explains the apparent paradox between scale independence and hierarchical linkages. Incorporating more hierarchical levels results in an increase in complexity or entropy of the network as a whole, but at a nonlinear rate. Complexity increases as a power α of the number of levels in the hierarchy, with $\alpha < 1$ and usually ≤0.6. However, algebraic connectivity decreases at a more rapid rate. Thus, the ability to infer one part of the hierarchical network from other level decays rapidly as more levels are added. Relatedness among system components decreases with differences in scale or resolution, analogous to distance decay in the spatial domain. These findings suggest a strategy of identifying and focusing on the most important or interesting scale levels, rather than attempting to identify the smallest or largest scale levels and work top-down or bottom-up from there. Examples are given from soil geomorphology and karst flow networks.

1. Introduction

In the geosciences we deal with temporal scales ranging from near instantaneous (e.g., fluid dynamics) to the entire >4 billion year span of Earth history. We work at spatial scales ranging from particles (and occasionally molecules) up to planetary. Even within a given domain, such as soils or fluvial systems, the subject matter incorporates much of that scale range. Because the same constructs—be they rules or tools—do not apply across the entire range of scales, we are confronted with the problem of scale linkage—that is, how to transfer knowledge, information, relationships, and representations among scales where the rules and tools are not always the same. The purpose of this paper is to use graph theory and network analysis to explore changes in overall system complexity as the range of scales considered is broadened, to identify the most promising general strategies for addressing scale linkage. Specifically, the goal is to determine the rate at which relatedness among components declines as additional hierarchical scale levels are considered.

Both spatial and temporal scales are often represented as hierarchies, as explicitly recognized in applications of hierarchy theory and hierarchical techniques of spatial analysis. The hierarchical nature of scale in Earth systems is also often implicit. In some cases the hierarchies are functional and spatially nested, and therefore theoretically unambiguous, such as the hierarchy of hillslopes and zero-order drainage basins to first order to n^{th} order catchments, to subcontinental drainages. In other cases the hierarchies are additive and equally clear (e.g., individuals, populations, communities, ecosystems, landscapes). In still other cases the hierarchical levels are based on conceptual models and may have fuzzy or arbitrary boundaries, but are widely used and generally agreed upon within a research community, and not controversial (e.g., the widely used pedological hierarchy originally presented by Dijkerman, 1974). Finally, in some cases hierarchies are imposed by nested scales or resolutions of maps or mapping programs or pixel sizes.

This paper is concerned with distance in a scale hierarchy in a way analogous to geographical distances in Tobler's (1970) "first law of geography." This states that everything is related to everything else, and that near things are more related than far things. While neither part of Tobler's first law is literally true everywhere and always, both are useful generalizations (Phillips, 2004). The latter part of Tobler's first law (TFL2) relates to the idea that the closer phenomena are, the more related or similar they are likely to be. This is a fundamental, venerable concept of geography, often expressed in terms of distance decay and spatial dependence, and predates Tobler (1970). The toolbox of spatial analysts is well stocked with methods for analyzing and modeling

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spatial dependence. A logical corollary of TFL2 is that phenomena that vary or operate over similar spatial scales are more related than those that manifest over more different scales. This notion is formalized in hierarchy theory (HT).

Haigh (1987) was apparently the first to propose HT as a tool for addressing scale linkage in geomorphology. Analytical applications (as opposed to as a pedagogic or heuristic device) are relatively rare, but a few examples exist in geomorphology (e.g., Dikau, 1990; Parsons and Thoms, 2007; Yalcin, 2008), and many more in landscape ecology (see reviews by O'Neill et al., 1986; Pelosi et al., 2010; Reuter et al., 2010). HT is a key conceptual and operational tool for addressing scale linkage in a geographic information systems (GIS) context (Dikau, 1990; Wu, 1999; Wu and David, 2002) and in geography more generally (Meentemeyer, 1989). Albrecht and Car (1999), for example, outlined a hierarchy-theory based approach for scale-sensitive GIS analysis. Hierarchy theory was applied to the problem of choosing and integrating among scales in the form of multiresolution remotely sensed data by Phinn et al. (2003), who used their method to analyze coastal landscapes. Bergkamp (1998) applied HT to analysis of runoff and infiltration interactions with vegetation & microtopography, and Yalcin (2008) showed that a hierarchy-based method for mapping landslide susceptibility produced more realistic results than alternative methods. Hierarchy theory has also been applied to the detection of landscape boundaries in ecology (Yarrow and Salthe, 2008), and to cross-scale modeling of nutrient loading in hydrologic systems (Tran et al., 2013).

Hierarchy theory is based on a nested structure of scales or resolutions. At a given level i, patterns and dynamics are affected by factors and processes operating at that level, at one level below (finer scale; $i - 1$), and at one level above (coarser scale; $i + 1$). Scales two or more levels away from the scale of observation involve factors that operate too rapidly or at too fine a resolution; or too slowly or at too coarse a scale, to be observed at i, or effects are entirely mediated by intermediate levels. HT is sometimes misunderstood as a tool or conceptual framework potentially enabling seamless linkage across the entire range of relevant scales. Actually, HT implies that scale linkage must be stepwise; as one ascends or descends the “scale ladder”, new factors and processes become relevant and others cease to be relevant.

The problem of scale linkage has been formally acknowledged for more than half a century. In 1965 Schumm and Lichty (1965) published their famous paper on the relationship between temporal scale and (in) dependence of variables and factors in geomorphology. The same year, Haggett (1965) articulated the broader problem of scale linkage. Phillips (1986, 1988) later derived a formal theoretical basis to support Schumm and Lichty's arguments.

If the “rules” concerning processes and functional relationships were constant across scales, then scale linkage would be mainly a technical issue. Such problems crop up, and remain challenging, with respect to issues such as multiple-resolution models, upscaling, and downscaling. However, the rules are typically not constant across scales, which is consistent with intuition, empirical evidence, and dynamical systems theory (Phillips, 1986, 1988, 2005). Can, for instance, the global biogeography of ants shed light on the biogeomorphic impacts of spatial foraging strategies or nest site selections of particular ant species (or vice-versa)? Can the mechanics of flow shear stress acting on a gravel particle on a stream bed explain the long term evolution of fluvially-dissected landscapes (or vice-versa)?

If TFL2 holds in the scale domain, then near scale levels are more related than those farther apart. This implies that as hierarchical scale levels become increasingly distant, then the dynamics at those scales become increasingly disconnected.

2. Theory

Assume that a given phenomenon of interest is manifested or influenced at a hierarchy of scale levels i, $i = 1, 2, \ldots, q$. For example, flow and sediment dynamics in a stream channel are influenced by processes and responses occurring at scales ranging from fluid dynamics to evolution of large drainage basins. Likewise, environmental carbon dynamics are controlled by processes occurring at scales ranging from the molecular to planetary. The scale of interest or observation is denoted as x, $x \in q$, with $1 \le x \le q$. $S(x)$ is the system state or condition at scale or level x, and $F_i(x)$ indicates the effects of processes or controls at level i manifest at x. Thus

$$S(x) = \sum_{i=1}^{q} F_i(x) \tag{1}$$

Denoting the probability of observing effects of a given scale i at the scale of observation x as $p[F_i(x)]$, then

$$p[F_i(x)] \sim f|x - i| \tag{2}$$

This is a scale hierarchy analog of distance decay, indicating that observation of effects from a given scale is partly a function of how closely situated those scales are in a hierarchy. $p[F_i(x)] = 1$ when $x = i$; otherwise $p[F_i(x)]$ is inversely related to the scale difference between x, i.

Hierarchy theory is based on the effects at any given level being observable only at adjacent levels. Thus, in this framework,

$$p[F_x(x)] = 1; 1 > p[F_{i-1}(x)], p[F_{i+1}(x)] > 0; \text{and } p[F_i(x)] = 0 \text{ otherwise.} \tag{3}$$

Hierarchy theory is based on a priori definition of hierarchy-based causality. However, hierarchies may not conform to this ideal, or knowledge may not be sufficient to implement this approach.

2.1. Graph theory approach

The analytical framework here is based on a notion of a system characterized by n key components or variables, which potentially affect, and are affected by, each other. This is conceptualized here as a simple, undirected, connected graph. Applications of graph theory in geomorphology are reviewed by Heckmann et al. (2015). These interactions occur at q hierarchical levels. They may repeat at successive scales–for example, vegetation, soils, landforms, and hydrology all mutually influence each other at scales ranging from a patch or pedon up through landscapes or even higher levels. In other cases the key variables and interactions may change along the scale hierarchy. The links between hierarchical levels may vary, as illustrated below.

The overall Earth surface system (ESS) is thus an N-component network, $N = \Sigma n_q$, with each node or vertex v represented by one of the components or variables at one of the hierarchical levels, $v_i(j)$; $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, q$.

The network can be depicted as a graph with N nodes and m edges or links representing the connections between the components. The graph adjacency matrix A has entries of 1 if the row and column components are connected and 0 otherwise, with zeroes on the diagonal. Components or nodes of this simple, undirected graph are considered connected if they mutually influence each other. The number of edges associated with a node is its degree.

2.2. Network complexity

A standard measure of graph complexity in algebraic graph theory is the spectral radius, defined as the (real part of the) largest eigenvalue of the graph adjacency matrix A, which has N eigenvalues λ, such that $\lambda_1 \ge \lambda_2 \ge \ldots \ge \lambda_N$. Therefore the spectral radius $= \mathrm{re}(\lambda_1)$ (henceforth simply λ_1, for brevity's sake).

Other approaches to measuring graph/network complexity are reviewed by Mowshowitz and Dehmer (2012), with a focus on entropy based measures. Some of these entropy measures are directly related to the graph eigenvalues (Geller et al., 2012; Mowshowitz and Dehmer, 2012). Here the simple relationship indicated by Geller et al. (2012) is