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An integral image approach to performing multi-scale topographic position analysis

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ABSTRACT

Digital elevation model (DEM) derived measures of terrain ruggedness and relative topographic position are useful parameters for automated landform classification and are widely applied in soils, vegetation, and habitat mapping. These elevation residual attributes are inherently scale dependent because they are defined in the context of a local neighborhood. Several previous studies have focused on assessing the multi-scale properties of elevation residuals based on varying roving window sizes, DEM grid resolution resampling, and hierarchical object-based methods. The computationally intensive nature of large-window DEM filtering has previously prevented the application of the varying roving window size approach from being applied to study the scaling properties of the terrain ruggedness and topographic position at broader regional scales.

This paper explores the use of an integral image based approach to measuring the common relative topographic position metric deviation from mean elevation (*DEV*). The approach was applied to a large DEM of an extensive and heterogeneous region in eastern North America. Compared with traditional image filtering techniques, the integral image approach was extremely efficient for calculating *DEV*, enabling a fine-resolution multi-scale analysis of elevation residuals. A method is described to allow for the measurement of *DEV* at optimal scales for each grid cell within wide spatial scale ranges. A novel technique is also developed for visualizing the scaling characteristics of topographic position using color composite imagery.

1. Introduction

Topography is the general character of rough, complex, and irregular shaped surfaces. The concept of topography is most commonly applied to the Earth's uneven surface (Huggett and Cheesman, 2002) and is the focus of research in geography and geology and their related fields of cartography, geodesy, geomorphology, and hydrology. This wide interest in studying Earth's topography is due to the partial control it has over the abundance and distribution of energy, water, nutrients, and sediments within landscapes (Lindsay and Rothwell, 2008). Topography is therefore strongly linked to environmental phenomena involving the distribution of flora and fauna and exerts substantial influence over spatial variability in climate (Böhner and Antonic, 2009).

Geomorphometry, also known as digital terrain analysis, is the field that is concerned with quantifying the Earth's topography (Pike, 2000; Pike et al., 2008). Much of modern geomorphometry is focused on extracting information from DEMs. This process usually involves deriving topographic parameters from DEMs, including measures of local surface shape (e.g. slope gradient and curvature), orientation (i.e. slope aspect), and the related concepts of ruggedness and relative topographic position (Gallant and Wilson, 2000). It is this last class of attributes that is the focus of this paper. Ruggedness refers to the roughness of a surface while relative topographic position refers to how elevated a location is compared with its surroundings. Ruggedness and topographic position are useful for landform classification (Riley et al., 1999; Tagil and Jenness, 2008) and therefore are also commonly applied in soils, vegetation, and habitat mapping (Jenness, 2004). They are also properties affecting the exposure or sheltering of locations (Yokoyama et al., 2002; Lindsay and Rothwell, 2008).

Terrain shape attributes, such as slope and curvature, are theoretically defined for any point within the landscape. For a surface that can be expressed in mathematical form, it is possible to measure these shape parameters precisely for any point on the surface. In practice, however, shape attributes are usually derived from a DEM using interpolated surfaces fitted to the elevations contained within the area of a 3 × 3 pixel roving window (Evans, 1984). This fact gives rise to a well-documented scale dependency (Chang and Tsai, 1991; Deng et al., 2007; Goodchild, 2011) that is typically viewed as a negative consequence of the way these terrain attributes are estimated. Ruggedness and topographic position by comparison are attributes that are inherently scale dependent because they can only be defined over an area. These attributes attempt to quantify the topographic character of a location within the broader context of its surroundings or local

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neighborhood. For example, a rocky outcrop situated on a valley bottom can be considered to be high in the landscape at shorter spatial scales but low-lying at a broader regional scale. The way that topographic position varies over a range of scales, i.e. the *scale signature* (Wood, 1996; Drăgut et al., 2011), can be viewed as valuable information for interpreting the structure of landscapes (Drăgut et al., 2011; De Reu et al., 2013).

Several approaches have been proposed for extracting this scale-variant topographic position information from DEMs including the use of a varying sized roving windows (e.g. Grohmann and Riccomini, 2009; De Reu et al., 2013), the resampling of DEM data to varying grid resolutions (e.g. Gallant and Dowling, 2003), and the use of a hierarchical object-based approach (e.g. Drăgut and Eisank, 2011). Scaling through resampling can result in generalization and loss of information due to smoothing (Deng et al., 2007; Drăgut and Eisank, 2011). The hierarchical object-based approach shows promise but adds the complexity and potential subjectivity involved in the object delineation process. Perhaps the most straightforward of the available techniques, the application of varying roving window sizes (i.e. raster spatial filtering) has the main disadvantage of being extremely computationally intensive (Deng and Wilson, 2008; Grohmann and Riccomini, 2009), which can make its application with large DEM datasets and regional-scale analyses impractical. This characteristic of the varying window size multi-scaling approach usually implies that relatively few window sizes can be used to sample the scale signature (Deng et al., 2007) and window sizes are often selected in an ad-hoc manner rather than optimally. These computational issues led Grohmann and Riccomini (2009) to conclude that the use of the roving window approach should be restricted to local scale analysis while resampling (i.e. what they refer to as a search window and interpolation method) can be used for broader spatial scales.

In the field of computer vision and graphics the problem of the computational inefficiency of large-window image filtering has been dealt with through the development of the integral image data transformation (Crow, 1984) and its more recent widespread application (Viola and Jones, 2001). The purpose of this study is to assess the use an integral-image based approach for performing multi-scale topographic position analyses on DEMs.

2. Background

2.1. Measures of ruggedness and topographic position

Gallant and Wilson (2000) provide a comprehensive review of existing indices for measuring ruggedness and topographic position, which they term *elevation residuals*. Elevation residuals are topographic indices derived from DEMs using spatial filtering techniques (i.e. a roving window of radius r is centered on each grid cell in the DEM) to quantify the spatial pattern of topographic position or ruggedness within the context of a surrounding area. Gallant and Wilson (2000) suggest defining neighborhoods with circular shaped windows, although square windows are more commonly used in practice. These terrain attributes are based on characteristics of the statistical frequency distribution of the elevations within neighborhoods defined by the roving window. All of the elevation residuals utilize location parameters (measures of central tendency) and/or scale parameters (measures of spread).

The two most common measures of ruggedness include the elevation range, or local relief (*LR*), and the standard deviation of elevation (s). *LR* is defined as the difference in elevation between the window maximum and minimum elevation values (Gallant and Wilson, 2000; Gallant et al., 2005; Liu, 2008). This measure is particularly sensitive to the elevation outliers and Gallant and Wilson (2000) caution that spatial patterns of *LR* can include abrupt changes where peaks fall in and out of the roving window. The elevation standard deviation is by comparison less sensitive to the extremes in elevation and is thought to better represent surface roughness properties (Evans, 1984; Klinkenberg, 1992). In the restricted case of a 3 × 3 window size, the terrain ruggedness index of Riley et al. (1999) is equivalent to s and is commonly calculated using GIS software.

Several terrain attributes are used to quantify the spatial pattern of topographic position, but the most common include elevation percentile (*EP*), difference from mean elevation (*DIFF*), and deviation from mean elevation (*DEV*). *EP* is the percentage of the cells within a roving window that are lower than the center pixel's elevation (Gallant and Wilson, 2000). It has a natural range from 0–100% and is relatively robust against elevation outliers. *DIFF* is the difference between the window center's elevation and its mean elevation (Gallant and Wilson, 2000; Weiss, 2001). It has the same units as elevation and is either positive, indicating an elevated location, or negative, indicating a low-lying position. *DEV* is a unitless measure of topographic position and is calculated in the same way as *DIFF* except that the elevation difference is normalized by s, such that:

$$DEV(D) = \frac{z_0 - \bar{z}_D}{s_D} \tag{1}$$

where D is the size of the window, z_0 is the elevation of the window center cell, and $\bar{z}_D$ is the window mean elevation. D is measured either in map units or grid cells. Since pixel-centered roving windows must have odd-numbered dimensions (3, 5, 7, etc.), it is more convenient to use the window half-size, r, where $D = 2r + 1$. The series $r = 1, 2, 3, \ldots$ therefore describes the series of square pixel-centered windows, such that $r = 1$ denotes a 3 × 3 roving window, $r = 2$ denotes a 5 × 5 window, and so on.

While *DEV* is essentially the spatial pattern of local *z*-scores, this fact does not imply that the index can be used to determine the statistical probability of a particular value occurring (i.e. outlier detection) because elevation distributions are often non-Gaussian. Nonetheless, values of *DEV* do tend to lie well within the range −3.0 to 3.0. Unlike *DIFF*, *DEV* is a measure of relative topographic position that is scaled by the local ruggedness. This characteristic is particularly useful in applications involving heterogeneous landscapes (De Reu et al., 2013).

2.2. Integral images and spatial filtering

An integral image (I), also known as a summed area table, is a simple data transformation used to efficiently measure the sum of all values within rectangular sub-sets of a raster grid (Crow, 1984). A pixel value in I is the sum of the pixel values in the input image within the rectangular region defined by the pixel and an image corner, usually the upper left-hand corner, i.e. the image origin coordinate (0, 0) (Fig. 1A). I is similar to a two-dimensional cumulative distribution function. After the integral image transform is applied, the sum of a rectangular neighborhood centered on a pixel is computed using three mathematical operations (Fig. 1B) regardless of the neighborhood size. The mean value of any sub-region is then calculated by dividing the neighborhood total value by the number of pixels within the sub-region. An integral image based approach therefore enables the efficient calculation of many common image filtering operations using very large window sizes in a way that is impractical using traditional filtering methods.

Integral images make the calculation of *DIFF* trivial even when using very large window sizes. Importantly, a single integral image can be used to efficiently calculate the elevation residual not just for a single scale (window size) but rather for an entire range of scales. There is no need to recalculate I for each window size. It is this characteristic that enables an integral image based multi-scale approach to analyzing ruggedness and relative topographic position.

In addition to *DIFF*, it is also possible to calculate *DEV* using an integral image based approach. To do so requires the calculation of the first and second power integral images I and I^2. A second-power integral image is derived from squared input values. Given the sum and squared-sum values within a window, the standard deviation is estimated using one of the common single-pass algorithms. It is also possible to calculate skewness and kurtosis for large windows with this