Computers and Mathematics with Applications 64 (2012) 2744–2755

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

# A new exact solution for pricing European options in a two-state regime-switching economy

Song-Ping Zhu *,[1], Alexander Badran, Xiaoping Lu

*School of Mathematics and Applied Statistics, University of Wollongong, Australia*

ARTICLE INFO

*Article history:*
Received 12 January 2012
Received in revised form 10 August 2012
Accepted 17 August 2012

*Keywords:*
Regime switching
European options
Fourier transform
Fourier inversion
Black–Scholes model

ABSTRACT

In this study, we derive a new exact solution for pricing European options in a two-state regime-switching economy. Two coupled Black–Scholes partial differential equations (PDEs) under the regime switching are solved using the Fourier Transform method. A key feature of the newly-derived solution is its simplicity in the form of a single integral with a real integrand, which leads to great computational efficiency in comparison with other closed-form solutions previously presented in the literature. Numerical examples are provided to demonstrate some interesting results obtained from our pricing formula.
© 2012 Elsevier Ltd. All rights reserved.

## 1. Introduction

It is well known that the classical Black–Scholes model with constant volatility does not fully reflect the stochastic nature of financial markets. Consequently, there is a need for more realistic models that better reflect random market movements. One such formulation is a model with regime switching, in which the key parameters of an asset depend on the market mode (or "regime") that switches among a finite number of states. From an economic perspective, regime-switching behavior captures the changing preferences and beliefs of investors concerning asset prices as the state of a financial market changes. Since being introduced by Hamilton [1], there has been a growing body of empirical evidence suggesting that the distributions of asset returns in some cases are better described by a regime-switching process (see [2–7]).

Pricing financial derivatives with regime-switching models has been discussed in literature. Bollen [8] presented a lattice-based method for pricing both European-style and American-style derivatives. Like other lattice-based numerical approaches adopted to price financial derivatives without the assumption of regime switching, Bollen's approach is financially intuitive and easy to implement. However, for European-style derivatives under the assumption of only two economic states, most researchers have focused their attention on developing closed-form exact solutions. Naik [9] was the first to discuss pricing and hedging European-style options when the volatility of the risky asset is assumed to randomly jump between two states. He found an exact closed-form pricing formula in the form of a double integral for an arbitrary security with a given payoff function. Di Masi et al. [10] discussed mean–variance hedging for European options where the drift rate and volatility are driven by a regime-switching process. Herzel [11] argued that a closed-form solution for a European contingent claim can be found in terms of a "basis" option. But, he only wrote down the partial differential equation (PDE) that the option prices under a two-state regime-switching model must satisfy once the value of the "basis" option has been found, without actually solving the derived PDE. Guo [12] presented a closed-form formula for the arbitrage-free price of a European call option in a two-state economy. The result found by Guo [12] is more general than that found

* Corresponding author. Fax: +61 2 42214845.
*E-mail address:* spz@uow.edu.au (S.-P. Zhu).

[1] An adjunct Tang-Au-Qing Chair Professor of Jilin University, China.

0898-1221/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2012.08.005

by Naik [9] as the drift rate, volatility and continuous dividend yield are all assumed to be dependent upon the economic state. Buffington and Elliot [13,14] showed how the governing PDEs are formed for European-style options and presented a closed-form solution for this problem through the derivation of the characteristic function of the occupation times in each state. However, if one adopts their formula, a Fourier inversion must be performed numerically. Fuh et al. [15] claimed that Guo [12] made an error, and they too presented a closed-form formula in a very similar form. Once the probability density function of the occupation time in each state is found explicitly, as done in [15], it is not surprising that the European-style option prices for a two-state regime-switching economy can simply be written in a closed form as a discounted expectation of the terminal payoff under the risk-neutral measure. More recently, Sepp and Skachkov [16] adopted a similar approach to the one used in this paper, finding a two-branch solution to the PDEs associated with two-state regime-switching for a European call option in the Laplace space. However, they did not perform the Laplace inversion analytically and resorted to the use of a robust numerical scheme for the calculation of option values.

Unfortunately, all existing formulae are written in the form of either a double integral, as a direct result of taking the discounted expectation, or a closed-form solution in a transform space such as the Laplace space; no one has managed to show that a closed-form solution can be written in the form of a single integral with real integrand. Reducing the final form of the closed-form solution from a double integral to a single integral comprised of elementary functions not only simplifies the appearance of the formula, but also enhances the computational efficiency if numerical values need to be produced. The contribution of this paper is to provide a new closed-form formula to value European options in a two-state regime-switching economy. This is achieved through an exact solution to the PDE system for a European put option found via the Fourier transform method. A key feature of our new formula is that we have successfully performed Fourier inverse transform analytically and thus produced a final pricing formula containing a single integral of a real-valued function. Therefore, in comparison with other approaches in the literature, our new formula displays advantages in computational efficiency and accuracy.

The rest of the paper is organized as follows. In Section 2, the asset price dynamics in a regime-switching economy are briefly described, followed by a detailed description of the newly found closed-form formula for the value of a European put option. In Section 3, numerical examples are given for the purpose of illustration, followed by the conclusions in Section 4. Any mathematical derivations that are not immediately needed in the main body of the paper, yet are important for readers who may be interested in the details of derivation, are left to the Appendices.

## 2. New solution

We model a European put option in a regime-switching economy where the drift rate and volatility are subject to random shifts between two states. The asset-price dynamics in a regime-switching economy have been described previously in [12–15]. However, for completeness, we start this section by briefly describing them as well.

The fluctuations of an asset are assumed to follow a stochastic process described by the stochastic differential equation

$$dS_t = \mu_{X_t} S_t dt + \sigma_{X_t} S_t dW_t \tag{1}$$

where $X$ is a continuous-time Markov chain with a finite state space. The drift rate, $\mu_{X_t}$, and the volatility rate, $\sigma_{X_t}$, of the asset are functions of $X_t$. $W$ is the standard Wiener process and the processes $X$ and $W$ are assumed to be independent. For each state, the drift rate and the volatility rate are assumed to be given constants. Furthermore, it is assumed that the volatility rates are distinct (i.e. $\sigma_{X_s} \neq \sigma_{X_t}$ if $X_s \neq X_t$).

In this paper, we assume $X$ is a two-state Markov chain which jumps between two states,

$$X_t = \begin{cases} 1, & \text{when the economy is in a state of growth} \\ 2, & \text{when the economy is in a state of recession.} \end{cases}$$

The transition between states occurs as a Poisson process, i.e.

$$P(t^*_{jk} > t) = e^{-\lambda_{jk} t}, \quad j, k = 1, 2, \ j \neq k$$

where $\lambda_{jk}$ is the transition rate from state $j$ to state $k$ and $t^*_{jk}$ is the time spent in state $j$ before entering state $k$.

The market price of risk associated with a change in state is not uniquely determined since the market is incomplete. In this paper, we assume that the risk associated with a regime switch is diversifiable and therefore not priced. Naik [9] demonstrates that this assumption does not result in a loss of generality, since one only needs to adjust the rate parameters of the transition process to account for non-diversifiable risk. Under this assumption, a system of coupled Black–Scholes equations for the value of a European put option can be derived (cf. [13,14]), with the movement of the underlying asset being described by Eq. (1), as

$$\begin{cases} \dfrac{\partial V_1}{\partial t} + \dfrac{1}{2}\sigma_1^2 S^2 \dfrac{\partial^2 V_1}{\partial S^2} + rS\dfrac{\partial V_1}{\partial S} - rV_1 = \lambda_{12}(V_1 - V_2) \\ V_1(0, t) = Ee^{-r(T-t)} \\ \lim\limits_{S\to\infty} V_1(S, t) = 0 \\ V_1(S, T) = \max\{E - S, 0\} \end{cases} \tag{2}$$

Download English Version:

https://daneshyari.com/en/article/468455

Download Persian Version:

https://daneshyari.com/article/468455

Daneshyari.com