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Computer simulation of MHD blood conveying gold nanoparticles as a third grade non-Newtonian nanofluid in a hollow porous vessel

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ABSTRACT

In this paper, heat transfer and flow analysis for a non-Newtonian third grade nanofluid flow in porous medium of a hollow vessel in presence of magnetic field are simulated analytically and numerically. Blood is considered as the base third grade non-Newtonian fluid and gold (Au) as nanoparticles are added to it. The viscosity of nanofluid is considered a function of temperature as Vogel's model. Least Square Method (LSM), Galerkin method (GM) and fourth-order Runge–Kutta numerical method (NUM) are used to solve the present problem. The influences of the some physical parameters such as Brownian motion and thermophoresis parameters on non-dimensional velocity and temperature profiles are considered. The results show that increasing the thermophoresis parameter (N_t) caused an increase in temperature values in whole domain and an increase in nanoparticles concentration just near the inner wall of vessel. Furthermore by increasing the MHD parameter, velocity profiles decreased due to magnetic field effect.

1. Introduction

The flows of non-Newtonian fluids have important role in several industrial and engineering processes, for example, petroleum drilling, paper production, glass blowing, plastic sheet formation, the extrusion of polymeric fluids and melts, biological solutions, paints, asphalts and glues etc. Nanofluids appear to have the potential to significantly increase heat transfer rates in a variety of areas such as industrial cooling applications, nuclear reactors, transportation industry, micro-electromechanical systems, heat exchangers, chemical catalytic reactors, fiber and granular insulation, packed beds, petroleum reservoirs and nuclear waste repositories and biomedical applications [1].

In review of its importance, the flow of Newtonian and non-Newtonian fluids through two infinite parallel vertical plates has been investigated by numerous authors. Ellahi et al. [1] used Reynolds and Vogel's model for viscosity of non-Newtonian nanofluid in a porous medium, also he and his co-workers [2] studied the effect of magneto-hydrodynamic (MHD) on non-Newtonian nanofluid flow between two coaxial cylinders. Non-Newtonian fluids have many applications in biomedical science. For example blood can be a non-Newtonian fluid which its physical properties are presented by Abdel Baieth [3]. Ogulu and Amos [4] modeled pulsatile

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Nomenclature

A, B	constants in viscosity function
B_r	Brownian diffusion constant
c	pressure gradient
c_i	constant coefficients
D_b	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
g	gravitational vector
G_r	thermophoresis diffusion constant
k	permeability
M	MHD parameter
N_b	Brownian motion parameter
N_t	thermophoresis parameter
P	porosity parameter
R_1	inner radius
R_2	outer radius
$R(x)$	residual function
V	velocity vector
$W(x)$	weighted function

Greek symbols

α, β	material moduli
β_T	volumetric expansion coefficient
Λ	third-grade parameter
λ_r	retardation time
μ	viscosity
ϕ	nanoparticle volume fraction
θ	temperature
ρ_f	density of the base fluid
ρ_p	density of the nanoparticles
φ	porosity

blood flow in the cardiovascular system employing the Navier–Stokes equation. Investigation on non-Newtonian fluid flow in microchannels and flow characteristics of deionized water and the PAM solution over a wide range of Reynolds numbers has been done by Tang et al. [5]. Yoshino et al. [6] presented a new numerical method for incompressible non-Newtonian fluid flows based on the lattice Boltzmann method (LBM). Their simulations indicate that the method can be useful for practical non-Newtonian fluid flows, such as shear-thickening (dilatant) and shear-thinning (pseudo-plastic) fluid flows. Xu and Liao [7] studied the unsteady magnetohydrodynamic viscous flows of non-Newtonian fluids caused by an impulsively stretching plate by means of homotopy analysis method to investigate the effect of integral power-law index of these non-Newtonian fluids on the velocity. In an experimental study Kumar et al. [8] investigated the effect of gold nanoparticles in blood from biomedically view point.

Many studies are focused on second and third grade non-Newtonian fluids which some of them are presented in this section. Modeling and solution of the unsteady flow of an incompressible third grade fluid over a porous plate within a porous medium is investigated by Aziz et al. [9]. They considered that the fluid is electrically conducting in the presence of a uniform magnetic field applied transversely to the flow. The magnetohydrodynamic (MHD) flow due to non-coaxial rotations of a porous disk, moving with uniform acceleration in its own plane and a second grade fluid at infinity is examined by Asghar et al. [10]. Keimanesha et al. [11] solved the problem of a third grade non-Newtonian fluid flow between two parallel plates by Multi-step Differential Transformation Method (Ms-DTM). The influence of third grade, partial slip and other thermophysical parameters on the steady flow, heat and mass transfer of viscoelastic third grade fluid past an infinite vertical insulated plate subject to suction across the boundary layer has been investigated by Baoku et al. [12]. Furthermore, Hayat et al. [12–15] completely discussed about the treatment of third grade non-Newtonian fluids in different applications.

There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and Least Square are examples of the WRMs. Stern and Rasmussen [16] used collocation method for solving a third order linear differential equation. Vaferi et al. [17] have studied the feasibility of applying of Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Hendi and Albugami [18] used Collocation and Galerkin methods for solving Fredholm–Volterra integral equation. Recently Least Square Method is introduced by Aziz and Bouaziz [19] and is applied for a predicting the performance of a longitudinal fin [20]. They found that least squares method is simple compared with other analytical methods. Shaoqin and Huoyuan [21] developed and analyzed least-squares approximations for the incompressible magneto-hydrodynamic equations. Recently Hatami et al. [22] and Sheikholeslami et al. [23] applied LSM and CM on fin performance and nanofluid in porous channel respectively. Ellahi [24] used homotopy analysis method (HAM [25–30]) analytical solution of MHD non-Newtonian nanofluid in a pipe. In another study, Rashad et al. [31] investigated the natural convection of non-Newtonian nanofluid around a vertical permeable cone.

The main aim of this paper is to simulate the problem of heat and fluid flow analysis for non-Newtonian nanofluid in porous media of hollow vessel by least square, Galerkin and numerical methods. Also the effects of some parameters such as Brownian motion and thermophoresis parameters on velocity and temperature profiles are investigated.

2. Description of the problem

Consider a steady, incompressible, non-Newtonian nanofluid in presence of magnetic field and in the porous media of hollow vessel as shown in Fig. 1a. For considering the natural convection, the nanofluid's density, ρ should be defined [32]

$$\begin{aligned}\rho &= \phi\rho_p + (1-\phi)\rho_{f_0} \\ &\cong \phi\rho_p + (1-\phi)\{\rho_f(1-\beta_T(\theta-\theta_w))\}\end{aligned} \tag{1}$$

where ρ_{f_0} is the base fluid's density, θ_w, is a reference temperature, ρ_f is the base fluid's density at the reference temperature, β_T is the volumetric coefficient of expansion. Taking the