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Critical conditions for particle motion in coarse bed materials of nonuniform size distribution

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A R T I C L E I N F O

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A B S T R A C T

Initiation of particle motion in a bed material of nonuniform size distribution may be quantified by $(q_{ci}/q_{cr}) = (D_i/D_r)^b$, where q_{ci} is the critical unit discharge at which particle size D_i enters motion, q_{cr} is the critical condition for a reference size D_r unaffected by the hiding/exposure effects associated with nonuniform size distributions, i and r refer to percentiles of the distribution and b varies from 0 (equal mobility in entrainment of all particle sizes) to 1.5–2.5 (full size selective transport). Currently there is no generally accepted method for predicting the value of b. Flume and field data are therefore combined to investigate the above relationship. Thirty-seven sets of flume data quantify the relationship between critical unit discharge and particle size for bed materials with uniform size distributions (used here to approximate full size selective transport). Field data quantify the relationship for bed materials of nonuniform size distribution at 24 sites, with b ranging from 0.15 to 1.3. Intersection of the two relationships clearly demonstrates the hiding/exposure effect; in some but not all cases, D_r is close to the median size D_{50}. The exponent has two clusters of values: $b > 1$ for sites subject to episodic rain-fed floods and data collected by bedload pit trap and tracers; and $b < 0.7$ for sites with seasonal snowmelt/glacial melt flow regimes and data collected by bedload sampler and large aperture trap. Field technique appears unlikely to cause variations in b of more than about 0.25. However, the clustering is consistent with possible variations in bed structure distinguishing: for $b > 1$, sites with relatively infrequent bedload transport where particle embedding and consolidation could reduce the mobility of coarser particles; and, for $b < 0.7$, a looser bed structure with frequent transport events allowing hiding/exposure and size selection effects to achieve their balance. As yet there is no firm evidence for such a dependency on bed structure but variations in b could potentially be caused by factors outside those determining equal mobility or size selection but appearing to affect b in the same way.

1. Introduction

Full quantification of the conditions determining the initiation of particle motion in river bed materials with nonuniform size distributions has been constrained by a lack of field data. Publications are often based on data collected at just one or two sites and a general, fully quantified equation for predicting the critical conditions still remains to be agreed. This paper therefore assembles a wide range of flume and field data, both published and unpublished, to investigate the formula for linking the critical flow condition to particle size. Particular points that are addressed include:

- the difference between the critical conditions for bed materials with uniform and nonuniform size distributions;
- the rate of change of maximum particle size in motion with the applied flow condition; and
- the roles that bed structure and data collection technique play in determining the rate of change.

2. Background

2.1. Mobility relationship

Within a bed material of nonuniform size distribution, the stability of a particle is affected by the ratio of its size to a reference size. Particles larger than the reference size are relatively easier to move than the same particles in a uniform bed material because they project above the smaller sizes and experience a higher drag force (Egiazaroff, 1965; Fenton and Abbott, 1977); the pivoting angle through which they need to be tipped to begin moving may also be smaller (Li and Komar, 1986; Komar and Li, 1988). Particles smaller than the reference size are relatively more difficult to move than if they were in a uniform bed material because they are hidden behind larger particles and on average the pivoting angle is larger. Particles of the reference size are unaffected by the hiding/exposure effect and behave as if in a bed of uniform

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material. The overall hiding/exposure effect can be quantified as (Parker et al., 1982; Andrews, 1983)

$$\tau^*_{ci}/\tau^*_{cr} = (D_i/D_r)^x \quad (1)$$

where τ^*_{ci} is the critical value of the Shields dimensionless shear stress required to set in motion a particle of size D_i

$$\tau^*_{ci} = \tau_{ci}/(\rho_s - \rho)g\,D_i \quad (2)$$

and D_i is the size of particle median axis for which i % of the particles are smaller, τ_{ci} is the critical shear stress for D_i, ρ_s is sediment particle density, ρ is water density, g is the acceleration due to gravity and S is stream gradient. Subscript r refers to the reference particle size that is not affected by the hiding/exposure effect. This size must lie between the smaller and larger sizes and is often assumed to be D_{50}, although this has not yet been conclusively demonstrated (e.g., Komar, 1987). The exponent x indicates the rate of change of τ^*_{ci} with D_i and varies from -1 for equal mobility in entrainment (all particle sizes are mobilised at the same flow) to 0 for full size selection (individual size fractions have no influence on each other and the force required to initiate movement of a given size is equal to that required to move the same size in a bed composed of uniform material of that size). Field and flume measured values in the literature vary from -0.6 to -1 (Ashworth and Ferguson, 1989; Komar and Shih, 1992; Andrews, 1994; Ferguson, 1994; Petit, 1994; Mao et al., 2008).

Discharge data are more readily available than shear stress data for natural rivers and discharge-based equations analogous to Eq. (1) may therefore be proposed as a matter of convenience:

$$q_{ci}/q_{cr} = (D_i/D_r)^b \quad (3)$$

where q_{ci} is the critical unit discharge (discharge per unit width) for particles of size D_i and b varies according to the balance between equal mobility and full size selection (Bathurst, 1987). The critical unit discharge q_{cr} for the reference particle size D_r can be obtained from an equation for uniform material of the form (e.g., Schoklitsch, 1962; Bathurst et al., 1987)

$$q_{cr} = Kg^{0.5}D_r^{\;1.5}S^{-M} \quad (4)$$

This equation can be derived theoretically from the Shields equation using a flow resistance relationship but, to avoid uncertainty arising from the choice of flow resistance relationship, K and M can also be determined empirically. By analogy with the shear-stress based approach, equal mobility requires that $q_{ci} = q_{cr}$ and b is then zero. For full size selection, the discharge required to move a given size might be expected to be the same as that required to move that size in a bed of uniform material. Eq. (4) would apply and the value of b would be 1.5. However, because the flow resistances of uniform and nonuniform bed materials are not entirely equivalent, Eq. (4) is not a complete analogy of the Shields condition for full size selection. Ferguson (1994) demonstrates the equivalence of the shear stress- and discharge-based methods and shows how x and b can be related mathematically as a function of a flow resistance equation. If flow resistance is represented by

$$(1/f)^{0.5} = m(d/D_{50})^c \quad (5)$$

where f is the Darcy–Weisbach resistance coefficient and m and c vary with the channel and its resistance characteristics, then

$$b = (1+x)(c+1.5) \quad (6)$$

For uniform materials c disappears from this equation but for nonuniform materials it can vary from 1/6 to nearly 1 (Bathurst, 2002; Ferguson, 2007), implying that, for full size selection, b can potentially vary from 1.67 to nearly 2.5. The value of 1.5 indicated by Eq. (4) is therefore only an approximation to the minimum likely value.

2.2. Factors affecting the rate of change of critical flow condition with particle size

To date, no generally agreed method has been developed to enable x or b to be quantified predictively for a given site. Simple dependencies on particle shape (e.g., Carling et al., 1992; Gomez, 1994) or particle size distribution (D_{84}/D_{16}) (Bathurst, 1987), have not proved a satisfactory basis for determining the critical conditions. However, Kirchner et al. (1990) have suggested that bed-packing geometry (and within bed variability of critical shear stress) has a far greater effect. The following bed-scale processes and factors are therefore proposed for analysis of their effect on entrainment:

- Consolidation of the bed during periods of immobility increases the shear stress needed to move the particles; critical conditions for initiation and cessation of motion then differ (Reid and Frostick, 1984; Reid et al., 1985).
- Increasing the duration of stress history (i.e., flows capable of agitating the bed particles without necessarily moving them) increases antecedent bed stability; increasing the antecedent shear stress decreases antecedent bed stability (Haynes and Pender, 2007).
- Embeddedness (defined as the average fraction of a gravel particle's perimeter surrounded by sand or finer sediments Salent et al., 2006) particularly affects coarser particles (e.g., Raudkivi and Ettema, 1982) and may be expected to reduce their protrusion and mobility.
- Lateral and longitudinal sorting of fine bed material into patches (Lisle and Madej, 1992; Lisle, 1995; Paola and Seal, 1995) allows entrainment of fines at lower discharges than might be expected from equal mobility considerations for the full range of bed material sizes, thus producing an impression of selective entrainment.

Wilcock and Crowe (2003) plotted a relative critical shear stress τ_{ci}/τ_{c50} against D_i/D_{50} using flume data for a gravel bed with different proportions of added sand. For D_i/D_{50} less than unity the exponent of the resulting power law was 0.12, while for D_i/D_{50} greater than unity it was 0.67. This is the only study in the literature that finds a change in the exponent between the finer and coarser halves of the bed material size distribution. The authors do not offer an explanation for the variation but it needs to be tested for wider application.

2.3. Measurement techniques and their associated uncertainty

There is a range of approaches for the definition and measurement of the critical entrainment conditions. These are not necessarily consistent (e.g., Komar and Shih, 1992; Buffington and Montgomery, 1997) and it is still to be determined if the differences between them have a greater or lesser effect on the apparent parameter values of the entrainment functions (Eqs. (1) and (3)) than the natural controls themselves.

Wilcock (1988) describes the two main approaches to quantifying the critical conditions as:

(i) The reference method. For each particle size fraction a fitted relationship between measured shear stress and transport rate is back extrapolated to give, as the critical condition, the bed shear stress that corresponds to a small (arbitrarily defined) transport rate (e.g., Parker et al., 1982).
(ii) The competence method. The critical condition is defined by the maximum particle size observed to be moved by a flow of given shear stress, stream power or discharge.