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Dynamic relaxation in algebraic reconstruction technique (ART) for breast tomosynthesis imaging

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ABSTRACT

Background and objectives: A major challenge in Digital Breast Tomosynthesis (DBT) is handling image noise since the 3D reconstructed images are obtained from low dose projections and limited angular range. The use of the iterative reconstruction algorithm Algebraic Reconstruction Technique (ART) in clinical context depends on two key factors: the number of iterations needed (time consuming) and the image noise after iterations. Both factors depend highly on a relaxation coefficient (λ), which may give rise to slow or noisy reconstructions, when a single λ value is considered for the entire iterative process. The aim of this work is to present a new implementation for the ART that takes into account a dynamic mode to calculate λ in DBT image reconstruction.

Methods: A set of initial reconstructions of real phantom data was done using constant λ values. The results were used to choose, for each iteration, the suitable λ value, taking into account the image noise level and the convergence speed. A methodology to optimize λ automatically during the image reconstruction was proposed.

Results: Results showed we can dynamically choose λ values in such a way that the time needed to reconstruct the images can be significantly reduced (up to 70%) while achieving similar image quality. These results were confirmed with one clinical dataset.

Conclusions: With simple methodology we were able to dynamically choose λ in DBT image reconstruction with ART, allowing a shorter image reconstruction time without increasing image noise.

1. Introduction

Digital Breast Tomosynthesis (DBT) is an emerging radiological technique that is used for breast cancer imaging. It produces 3D information of the breast by acquiring a limited number of projections from a narrow angular range. This limited angular range of acquisition doesn't allow an isotropic spatial resolution: it is higher in the directions parallel to the detector than in the perpendicular axis. However, even with this anisotropy, DBT images can diminish the tissue superposition impact, which is the main phenomenon responsible for reducing sensitivity and specificity in Digital Mammography images [1].

In tomographic medical imaging, image reconstruction can be performed either by using analytical or iterative

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reconstruction algorithms. For DBT, many studies published in recent years [2–5] showed that iterative methods present many advantages over analytical algorithms, in particular because the first allow for physical constraints to be easily incorporated in the reconstruction process.

Iterative algorithms can be subdivided into two classes: algebraic and statistical [6]. In contrast to statistical algorithms, the algebraic methods ignore the statistical nature of the imaging process. However, on the other hand, algebraic algorithms have a faster convergence speed for DBT [7].

Algebraic Reconstruction Technique (ART) is an algebraic iterative image reconstruction method developed by Kaczmarz to solve linear equations systems. Its first application to medical imaging was done by Herman [8]. The first step in all algebraic reconstruction algorithms is the definition of a finite parameterization of the object, resulting in a discrete set of voxels. Let the whole volume be subdivided into J voxels and the linear attenuation coefficient for the jth voxel be denoted by u_j $(1 \le j \le J)$. Considering a detector with I bins, the number of photons detected in detector bin i is given by Eq. (1). Assuming that the number of rays is equal to the number of detector elements, the ith ray $(1 \le i \le I)$ is defined as the line segment connecting the x-ray source location with the center of the ith detector element. In addition, a_{ij} stands for the length of the section of the ith ray within voxel j, and d_i is proportional to the number of photons leaving the source toward the detector.

$$\bar{y}_i = d_i e^{-\sum_{j \in I_i} a_{ij} u_j} \tag{1}$$

Taking log-converted data, an alternative representation is Eq. (2).

$$y_i = \sum_j a_{ij} u_j \tag{2}$$

y_i can be seen as a hyperplane where the solution of u must lie. There are as many hyperplanes as projections and the solution of u must belong simultaneously to all hyperplanes, i.e., it should be on the intersection of all hyperplanes. If we have at least the same number of projections and image pixels, in the presence of noiseless data (ideal situation) the intersection of all hyperplanes is a single point and the solution is unique. However, this is not what happens with real data, which is not noise free and the number of projections is much lower than J, due to the limited acquisition time and angular coverage. So, image reconstruction becomes an ill-posed problem and achieving a reliable solution is challenging.

ART's iterative process starts with the definition of a first estimate of function u. This initial estimate will correspond to a vector in the space defined by the hyperplanes. The first iteration of the algorithm will correspond to the projection of this vector onto one of the hyperplanes, determining the point on the hyperplane closest to the point of the estimate. This new point will then be used as an estimate for the next iteration that will consist in projecting it onto other hyperplane. This procedure is repeated for all hyperplanes, hopefully forcing the estimate to converge to the desired solution [9].

The numerical expression that represents this operation is given by Eq. (3), where k is the iteration number:

$$u_j^k = u_j^{k-1} + \frac{y_i - \sum_l a_{il} u_l^{k-1}}{\sum_l a_{il}^2} a_{ij} \tag{3}$$

When noise is present in the acquired data, the assumption that a unique solution common to all projections exists is not true, since noise will incoherently shift every hyperplane [9]. In this situation, multiple intersections corresponding to partial solutions will probably arise, i.e., solutions that satisfy part of the constraints but not all of them. This will be a challenge to the iterative procedure, since it will not converge to a unique solution, but will be cyclically switching between partial solutions. This kind of behavior can be limited by introducing a relaxation parameter (λ^k) in the iterative expression used to update the solution u (Eq. (4)).

$$u_j^k = u_j^{k-1} + \lambda^k \frac{y_i - \sum_l a_{il} u_l^{k-1}}{\sum_l a_{il}^2} a_{ij} \tag{4}$$

In order to minimize the distance to the different partial solutions, this relaxation parameter must have a value between 0 and 1, forcing the final solution to converge to a point somewhere in the middle of the partial solutions. The applicability of this method to reconstruct DBT projections has been established [4,10–12], but the choice of the relaxation parameter value required in this algorithm is still an open issue. If we analyze the inverse problem using a geometrical approach, we realize that convergence speed is highly dependent on the relaxation parameter and the order by which the hyperplanes are considered to find the solution. Regarding the later dependence, the mathematical explanation relies on the fact that the more perpendicular two consecutive hyperplanes are, more new information is added to solve the problem. This is opposed to the situation where very close hyperplanes are used, both having almost the same information and thus not maximizing the contribution to the convergence of the solution [13].

The choice of the relaxation parameter value is critical to the quality and speed of image reconstruction using ART [8]. Although there is no generally established method to find the optimum value of λ, it is unanimously accepted that the choice of λ depends on the purpose of the image reconstruction. This parameter can be set during all the iterative process (constant) or it can be iteration dependent (dynamic). Typically, high constant values of λ allow faster convergence speed but also noisier reconstruction images. On the other hand, low constant values of λ allow smoother images but with slower convergence speed [8,13–18].

Although several studies on the optimization of λ have been published [8,13,15,16,19], including one by our group for Positron Emission Mammography image reconstruction [16], most of them are about using a constant λ. In addition, none of these studies address DBT data. In this paper we present a detailed study of the influence of a dynamic λ on the performance of the ART algorithm for DBT image reconstruction in order to reduce the time of reconstruction process. Using the results of this study we propose a methodology to adjust this parameter dynamically during the image reconstruction process, which results in decreased image reconstruction times and minimizes image noise.