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Continuous attractors of a class of neural networks with a large number of neurons

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A B S T R A C T

A class of simplified background neural networks model with a large number of neurons is proposed. Continuous attractors of the simplified model are studied in this paper. It contains: (1) When the background inputs are set to zero and the excitatory connections are in Gaussian shape, continuous attractors of the new network are obtained under some condition. (2) When the background inputs are nonzero and the excitatory connections are still in Gaussian shape, continuous attractors are achieved under some appropriately selected condition. (3) Discussions and examples are used to illustrate the theories developed.

1. Introduction

It is known that the background input plays very important roles in practical applications. For example, a gun shot may trigger a sudden motor response in the games. The same gun shot, however, may be unimportant if it sounds inside a theater. Another example is that the color of a visual stimulus may instruct the subject to perform different motor actions. A class of background neural networks model is proposed by Salinas [1] to analyze how the background controls the stability of the state in which all neurons fire at the same time. It shows that the background input has strong impact on neural activity (see [2–11]) that acts as a switch (see [12–15]) and allows the network to be turned on or off. Interestingly, the neural network in [1] can exactly exhibit continuous attractors when the excitatory connection weights are in Gaussian shape and some parameters are appropriately tuned.

In recent years, continuous attractors of neural networks have been studied extensively. They are important dynamical properties in neurobiological models studies. There is good evidence for continuous stimuli, such as orientation, moving direction, and the spatial location of objects could be encoded as continuous attractors [16–21]. For example, the memory of eye position is stored in an approximate line attractor space when some parameters are appropriately selected in [19]. When the instantiations of an object lie on a continuous pattern manifold, Seung [20] proposed that the object may be represented by a continuous attractor. That is to say, continuous attractors can model the manifold from which the patterns are drawn. Tang et al. [22] investigated the continuous attractors as limit cycle. However, activities of the above-mentioned models are characterized by line attractors or bump attractors and it is difficult for those to generate unimodal profiles of activity [3,5]. Recently, a class of recurrent neural networks can realize unimodal profiles of activity with some precisely tuned parameters in [17]. However, the authors did not take external inputs into consideration in [17] and did not analyze how the inputs affect continuous attractors.

Due to the complex network state equation in [1], it is hard to analyze some of its important properties such as continuous attractors that many brain theories have implicated in learning and memory. So far, there have been only a

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few simulations and exhaustive theoretical analysis for continuous attractors of the original background neural network model. Especially, when the background input is nonzero, the complexity of the analysis of continuous attractors in [1] grows rapidly as the division operation in the model. Therefore, these restrictions of the background neural network will limit its applications.

Inspired by Oja's idea [23], in this paper, a class of simplified background neural networks model with a large number of neurons is proposed through the use of Taylor's theorem. The division operation in the original network is replaced by the multiplication operation in the new one. The simplified network not only can produce unimodal activity but also switch between two different states under certain conditions. Compared with the model reported in [17], it can be seen that our model considers the input which can cause continuous attractors upward and wider. Thus, the proposed model can be seen as a promising alternative of the original one. In this paper, we mainly focus on the analysis for continuous attractors of the new model in two cases, i.e., the background inputs are set to zero and nonzero constants. Conditions for stability of equilibrium points of the model are obtained. Under the conditions, continuous attractors are achieved. Interestingly, we find that when the input is higher the network can produce persistent activity which can be switched on or off by the background input. Therefore, the input plays an important role such as a gating or context signal.

This paper is organized as follows. In Section 2, a class of simplified background neural networks model is proposed. In Section 3, continuous attractors of the new model are obtained under some conditions. In Section 4, further discussions are carried out. The simulation results are given in Section 5. Finally, conclusions are drawn in Section 6.

2. Preliminaries

In [1], Salinas proposed a class of background neural networks with a relatively large number of neurons as follows:

$$\tau \frac{dx(a)}{dt} = -x(a) + \frac{\left(\rho \int \tilde{w}(a,b)x(b)db + \tilde{h}(a)\right)^2}{s + \upsilon\rho \int x^2(b)db}, \tag{1}$$

where $\rho \geq 0$ is the density of neurons. $x(a)$ denotes the firing rate of the neuron a, where a and b act as the neuron's indices or labels, respectively. $\tilde{h}(a)$ denotes the background input. $\tilde{w}(a,b)$ corresponds to excitatory connection of two neurons a and b. $\upsilon \geq 0$ is the inhibitory connection. $\tau > 0$ is a time constant, $s > 0$ is a saturation constant.

Inspired by Oja's idea [23], a class of simplified background neural networks can be described by the following equation: (see Appendix for details)

$$\frac{dx(a,t)}{dt} = -x(a,t) + \left(\rho \int_{-\infty}^{+\infty} w(a,b)x(b,t)db + h(a)\right)^2 \left(1 - c\rho \int_{-\infty}^{+\infty} x^2(b)db\right), \tag{2}$$

for $t \geq 0$, where $c = \frac{\upsilon}{s}$ is a positive constant. $w(a,b) = \frac{\tilde{w}(a,b)}{\sqrt{s}}$, $h(a) = \frac{\tilde{h}(a)}{\sqrt{s}}$. For simplicity, the time constant τ is ignored.

The difference between the original background neural network model and the simplified one is that the division operation in the original model is replaced by the multiplication operation in the new one.

3. Continuous attractors

In this section, we will study continuous attractors of the network (2). Two stability conditions for (2) will be derived in two cases. i.e., all the inputs $h(a) = 0$ and $h(a) \neq 0$. In both cases, the simplified network can exhibit continuous attractors if the synaptic connections are in Gaussian shape and other parameters are appropriately tuned.

3.1. Case 1: $h(a) = 0$

In this case, when all the background inputs are set to zero, we can rewrite (2) as

$$\frac{dx(a,t)}{dt} = -x(a,t) + \left(\rho \int_{-\infty}^{+\infty} w(a,b)x(b,t)db\right)^2 \left(1 - c\rho \int_{-\infty}^{+\infty} x^2(b)db\right) \tag{3}$$

for $t \geq 0$. Suppose $x_{\max}(t)$ is a solution of (3).

It is easy to verify that zero is a stable equilibrium point of the following equation

$$\frac{dx_{\max}(t)}{dt} = -x_{\max}(t) + \pi\rho^2 w_{\max}^2 \sigma^2 x_{\max}^2(t)\left(1 - c\rho\sqrt{\pi}\sigma x_{\max}^2(t)\right) \tag{4}$$

for $t \geq 0$. Next, the stability of nonzero equilibrium points of (4) will be discussed.