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Pseudo almost automorphic solutions of some nonlinear integro-differential equations

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ABSTRACT

In this paper we discuss the existence and uniqueness of a pseudo almost automorphic solution of an integro-differential equation in a Banach space X. We achieve our results using the methods of fractional powers of operators and the Banach fixed point theorem. These results are new and complement the existing ones.

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1. Introduction

The concept of pseudo almost automorphic functions are a natural generalization of almost automorphic functions. The concept of the almost automorphic function was introduced by Bochner [1]. For more detail one can see the book by N'Guérékata [2] in which the author gave a very good overview of the theory of almost automorphic functions and their applications to differential equations. Almost automorphic solutions of various differential equations have been studied by many authors [2–6] and references therein. The concept of pseudo almost automorphy was suggested by N'Guérékata (see [2, page 40]) and developed by Xiao et al. [7]. The existence and uniqueness of pseudo almost automorphic solutions of differential equations have attracted the attention of many mathematicians in recent years [8–13,3,14].

Recently Xiao et al. [3] have shown the existence of an pseudo almost automorphic mild solution of the following differential equations

$$\frac{dx(t)}{dt} = A(t)x(t) + \bar{f}(t, x(t)), \quad t \in \mathbb{R}, \tag{1.1}$$

$$\frac{dx(t)}{dt} = A(t)x(t) + \bar{f}(t, x(t-h)), \quad t \in \mathbb{R}, \tag{1.2}$$

$$\frac{dx(t)}{dt} = A(t)x(t) + \bar{f}(t, x(t), x[\alpha(t, x(t))]), \quad t \in \mathbb{R}, \tag{1.3}$$

in a Banach space. The cases $A(t) = A$ and $A(t + p) = A(t)$ for some positive p have been studied by many authors (see for instance [1,7] and references therein).

Motivated by the works mentioned above, we will study in this paper the problem of existence and uniqueness of pseudo almost automorphic solutions of the following integro-differential equation in a complex Banach space X,

$$\frac{du(t)}{dt} + Au(t) = f(t, u(t), Ku(t)), \quad t \in \mathbb{R},\ u \in PAA(X),$$

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$$Ku(t) = \int_{-\infty}^{t} k(t-s)g(s,u(s))\mathrm{d}s, \tag{1.4}$$

where $f : \mathbb{R} \times X \times X \to X$, $g : \mathbb{R} \times X \to X$ and k satisfy $|k(t)| \le C_k e^{-bt}$ for $t \ge 0$ and C_k, b are positive constants. We denote $PAA(X)$ the set of all pseudo almost automorphic functions from $\mathbb{R}$ to X. Further, we assume that $-A$ is the infinitesimal generator of an analytic semigroup $\{T(t), t \ge 0\}$ and the function $f(\cdot, u(\cdot), Ku(\cdot)) \in BC(\mathbb{R} \times \mathbb{X} \times \mathbb{X}, X)$, where BC denotes the set of all bounded continuous functions.

In [15] Diagana et al. have shown the existence of pseudo almost periodic solution using fractional powers of operators of the following differential equation

$$\frac{\mathrm{d}u(t)}{\mathrm{d}t} + Au(t) = \bar{g}(t, u(t)), \quad t \in \mathbb{R}, \tag{1.5}$$

in a Banach space X, where $\bar{g} : \mathbb{R} \times X \to X$, is a jointly continuous function, $-A$, the generator of an analytic semigroup. Also Bahaj and Sidki [16] studied the existence of almost periodic solution of differential equation (1.5) using fractional powers of operators.

Because the concept of pseudo almost automorphic functions is pretty new, there is not much literature available on the pseudo almost automorphic solution of functional, delay and patrial differential equations. Many authors have shown the existence of a pseudo almost automorphic mild solution under the Lipschitz condition on the forcing term. Here we show the existence and uniqueness of a pseudo almost automorphic solution of (1.4) using the method of fractional powers of linear operators and the Banach fixed point principle. At the end we give an example to illustrate the abstract results.

2. Preliminaries

We denote by $BC(\mathbb{R}, X)$ the space of all bounded continuous functions from $\mathbb{R}$ to X. It is a Banach space with the supremum norm

$$\|u\|_\infty = \sup_{t \in \mathbb{R}} \|u(t)\|.$$

Consider $B(X, Y)$ is the set of all bounded linear operators from X to Y. This is also a Banach space with norm

$$\|A\|_{B(X,Y)} = \sup_{x \in X, x \neq 0} \frac{\|Ax\|_Y}{\|x\|_X}.$$

Similarly $BC(\mathbb{R} \times X \times X \to X)$ is the Banach space of bounded continuous functions with supremum norm. Now we give some necessary definitions.

Fractional powers of operators:

It is possible to define fractional powers of A if $-A$ is the infinitesimal generator of an analytic semigroup $T(t)$ in a Banach space and $0 \in \rho(A)$. We define the fractional power $A^{-\alpha}$, for $\alpha > 0$ by

$$A^{-\alpha} = \frac{1}{\Gamma(\alpha)} \int_0^\infty t^{\alpha-1} T(t)\mathrm{d}t.$$

It is well known that for $0 < \alpha \le 1$, $A^\alpha : D(A^\alpha) \subset X \to X$ is a densely defined closed linear operator. $D(A^\alpha) \supset D(A)$ is the domain of A^α which is dense in X. For $f \in D(A^\alpha)$ we define the norm by

$$\|f\|_{D(A)} = \|f\| + \|A^\alpha f\|.$$

This graph norm is equivalent to the $\alpha-$*norm* defined by $\|f\|_\alpha = \|A^\alpha f\|$. We denote X_α as the Banach space $D(A^\alpha)$ equipped with $\|\cdot\|_\alpha$. We observe that for $\alpha, \beta \ge 0$

$$A^{-\alpha-\beta} = A^{-\alpha}A^{-\beta}$$

and there exists a constant C such that $\|A^{-\alpha}\| \le C$ for $0 < \alpha \le 1$. The A^α is defined as the inverse of $A^{-\alpha}$. For a more detailed analysis on fractional powers of operators, the interested reader may consult [17].

Lemma 2.1. *Let $-A$ be the infinitesimal generator of an analytic semigroup $T(t)$. Then for $\alpha > 0$ and $0 \in \rho(A)$ we have,*

(i) *$T(t)A^\alpha x = A^\alpha T(t)x$ for every $x \in D(A^\alpha)$;*
(ii) *$T(t) : X \to D(A^\alpha)$ for every $t > 0$ and $\alpha \ge 0$;*
(iii) *for every $t > 0$ the operator $A^\alpha T(t)$ is bounded and*

$$\|A^\alpha T(t)\| \le M_\alpha t^\alpha e^{-\delta t};$$

(iv) *for $0 < \alpha \le 1$ and $x \in D(A^\alpha)$, we have*

$$\|(T(t)x - x)\| \le C_\alpha t^\alpha \|A^\alpha x\|.$$

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