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Constructal view of scaling laws of river basins

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Abstract

River basins are examples of naturally organized flow architectures whose scaling properties have been noticed long ago. Based on data of geometric characteristics, Horton [Horton, R.E., 1932. Drainage basin characteristics. EOS Trans. AGU 13, 350–361.], Hack [Hack, J.T., 1957. Studies of longitudinal profiles in Virginia and Maryland. USGS Professional Papers 294-B, Washington DC, pp. 46–97.], and Melton [Melton, M.A, 1958. Correlation structure of morphometric properties of drainage systems and their controlling agents. J. of Geology 66, 35–56.] proposed scaling laws that are considered to describe rather accurately the actual river basins. What we show here is that these scaling laws can be anticipated based on Constructal Theory, which views the pathways by which drainage networks develop in a basin not as the result of chance but as flow architectures that originate naturally as the result of minimization of the overall resistance to flow (Constructal Law).

Keywords: River basins; Scaling laws; Constructal theory

1. Introduction

Flow architectures are ubiquitous in nature. From the planetary circulations to the smallest scales, we can observe a panoply of motions that exhibit organized flow architectures: general atmospheric circulations, oceanic currents, eddies at the synoptic scale, river drainage basins, dendritic crystals, etc. Fluids circulate in all living structures, which exhibit special flow structures such as lungs, kidneys, arteries, and veins in animals and roots, stems, and leaves in plants.

Rivers are large-scale natural flows that play a major role in the shaping of the Earth's surface. River morphology exhibits similarities that are documented extensively in geophysics treatises. For example, Rodríguez-Iturbe and Rinaldo (1997) gave a broad list of allometric and scaling laws involving the geometric parameters of the river channels and of the river basins.

In living structures, heat and mass flow architectures develop with the purpose of dissipating minimum energy, therefore reducing the food or fuel requirement, and making all such systems (animals and "man+machine" species) more "fit," i.e., better survivors.

Constructal theory views the naturally occurring flow structures (their geometric form) as the end result of a process of area to point flow access optimization with the objective of providing minimal resistance to flow (see Bejan, 2000; Bejan and Lorente, 2004). The Constructal law first put forward by Bejan (1997) stated that "for a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed (global) currents that flow through it."

In the past few decades, extremal hypotheses (e.g. maximum sediment transporting capacity, minimum energy dissipation rate, minimum stream power, minimum

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Nomenclature

A	Area (m^2)
D	Drainage density (m^{-1})
F	Stream frequency (m^{-2})
H	Width of a construct (m)
$\hat{K}$, $\tilde{K}$	Dimensionless channel flow conductance
L	Stream length (m)
$\tilde{L}$	Dimensionless length (construct, channel)
N_i	Total number of streams of order i
n	Number of streams that are tributaries of each stream of the next order
R_B	Horton's bifurcation ratio
R_L	Horton's ratio of stream lengths
W	Channel width (m)
Φ	Aspect ratio, W/H

Subscripts

i	Order of a stream
s	Relative to stream
T	Total
ω	Order of the river basin
0	Relative to the elemental construct

Froude number) have been proposed as basis for deducing specific features of river basin morphology and dynamics (see for example the review by Huang and Nanson, 2000). Fractal geometry has also been used to describe river basin morphology (e.g., Rodríguez-Iturbe and Rinaldo, 1997; Cieplak et al., 1998). Fractals do not account for dynamics, hence are descriptive rather than predictive.

Because the same morphological laws may be deduced in apparently different contexts some authors have considered fluvial networks and basin geometries as canonical examples of equifinality, which is a concept invented by Beven (1993). Equifinality arises when many different parameter sets are equally good at reproducing an output signal. As pointed out by Savenije (2001), although these models may be based on physical relationships they are not unequivocal, and hence are not fit to be used as predictive models.

What is new with Constructal theory is that it unites geometry with dynamics in such a way that geometry is not assumed in advance but is the end result of an optimization procedure. Constructal theory is predictive in the sense that it can anticipate the equilibrium flow architecture that develops under existing constraints. In contrast with fractal geometry, self-similarity needs not to be alleged previously, but appears as a result of the constructal optimization of river networks. Moreover, Constructal theory shows that the hypotheses of minimum energy dissipation rate and minimum stream power are corollaries of the Constructal law under particular constraints (Reis, 2006).

The aim of this paper is twofold: to show how the scaling laws of river basins may be anticipated based on Constructal theory, and to present this theory to geomorphologists as a useful tool for the study of the

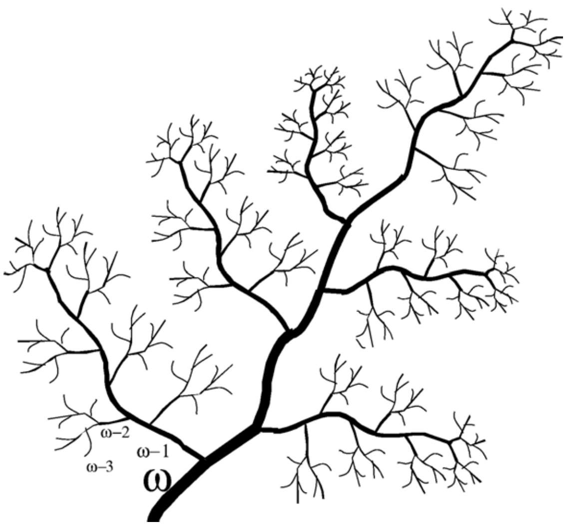

Fig. 1. River network with streams up to order ω.