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Scaling Properties of Feldspar and Quartz in Micro-images of Ideal Granites

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ABSTRACT: The properties of feldspar and quartze are studied in this article from a fractal point of view using gray-scale micro-images of granite samples collected at the Fangshan (房山) granite body in Hebei (河北) Province, China, which can be regarded as an ideal granite in the sense of Vistelius. We found that there exist power-law relationships between the eigenvalues of the gray-scale matrices and their ranks for the feldspar and quartz. The fractal model used here is a λ-R model similar to the N-λ model proposed by Qiuming Cheng in 2005. Meanwhile, we found that average variances for the gray-scale matrices of feldspar are larger than those of quartz on the same sections, and this may be useful for auto-identification of feldspar and quartz as well as other minerals.
KEY WORDS: fractal model, eigenvalue, granite, gray-scale matrix.

INTRODUCTION

Since the concepts of fractal and multifractal were introduced originally by Mandelbrot (1974, 1972), various fractal models have been developed and some of these have been widely used in various scientific fields for characterizing measures with scaling properties (Cheng, 2008, 2007, 2004, 1999a, b, 1997; Xu et al., 2007; Cheng et al., 1994; Evertsz and Mandelbrot, 1992; Chhabra and Sreenivasan, 1991; Schertzer and Lovejoy, 1991; Paladin and Vulpiani, 1987; Halsey et al., 1986; Badii and Politi, 1985, 1984; Frisch and Parisi, 1985; Grassberger, 1983; Hentschel and Procaccia, 1983).

Fractal models have also been extensively

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employed for distribution pattern analysis of minerals in space. Based on the Markov chain concept, Vistelius (1983) studied spatial arrangement probabilities of feldspar and quartz in granites resulting in the concept of "ideal granite". Based on fractal dimensions, the geometric characteristics of quartz were quantitatively analyzed and the changes of granite texture during crystallization and recrystallization were modeled in computer simulation experiments with subsequent use of results to characterize the effects of various geological processes on mineral particles (Gulbin and Evangulova, 2003). The frequency distribution and irregularity of sphalerite grains observed in mineral oscillatory banding were also quantitatively characterized by fractal models including perimeter-area (P-A), box-counting, and number-area (N-A) methods (Wang et al., 2007a). Fractal modeling has been demonstrated to be an effective way to quantify the degree of deformation including the irregularities of shapes and the frequency distribution of minerals in mylonite analysis (Wang et al., 2007b). Moreover, other nonlinear models have been used to study mineral growth and variance of geometric formation of mineral grains (L'Heureux and Fowler, 1996; Jamtveit et al., 1995; Wang and Wu, 1995; Wang and Merino, 1992; Jamtveit, 1991). In general, how to identify mineral grains presents a major λ modeling of the growth of minerals.

Cheng (2005a, b) pointed out that the frequency of large eigenvalues of a 2-D heterogeneity field follows the power-law: $N(\lambda_i>\lambda)\propto\lambda^{-\beta}$, which is used for decomposition of patterns (Li and Cheng, 2004), where λ_i is eigenvalue of the 2-D matrix for the heterogeneity field. While investigating gray-scale matrices of micro-images of feldspar and quartz in the so-called ideal granite (Xu et al., 2007; Vistelius et al., 1983; Vistelius, 1972), sample thin sections were found to possess similar power-law relationships related to the eigenvalues of a matrix. The motivation of this research is based on automatic mineral recognition under a microscope, which is thought to be significant to the research in several fields. Although there is still a long way to go to achieve this type of automatization, the preliminary results described in this article are believed to be useful.

EIGENVALUE-RANK FRACTAL MODEL

Assume that $\boldsymbol{A}=(a_{ij})_{n\times n}$ is an $n\times n$ matrix, λ_i (i=1, 2, …, n) is the eigenvalue of $\boldsymbol{A}$, and $|\lambda_i|$ is the absolute value of λ_i when λ_i is a real number or the modulus of λ_i when λ_i is a complex number. For simplicity, the values $|\lambda_i|$(i=1, 2, …, n) are supposed to be different from one another. Ordering $|\lambda_i|$(i=1, 2, …, n) from large to small, we get $|\lambda_{(1)}|$, $|\lambda_{(2)}|$, $\cdots$, $|\lambda_{(n)}|$ and define $R(\lambda_i)=R_i$ as the rank of $|\lambda_i|$(i=1, 2, …, n) if $|\lambda_i|=|\lambda_{(R_i)}|$. Then, if the eigenvalue $|\lambda|$ and its rank follow a power-law denoted by $|\lambda|\propto R^{-\alpha}(R>1)$, we say that the eigenvalues of matrix $\boldsymbol{A}$ are scale invariant according to their ranks, where α>0 is the invariance index.

Since $N(|\lambda_i|\geq|\lambda|)=R(|\lambda|)$, where $N(|\lambda_i|\geq|\lambda|)$ is the number of elements of the set $\{\lambda_i:|\lambda_i|>|\lambda|\}$, if there exits $N(|\lambda_i|\geq|\lambda|)\propto|\lambda|^{-\beta}$, i.e., $R=R(|\lambda|)\propto|\lambda|^{-\beta}$, then, $|\lambda|\propto R^{-\alpha}$, $\alpha=\beta^{-1}$. From this perspective, the λ-R model is similar to the N-λ model suggested by Cheng (2005a).

SAMPLING AND CALCULATION RESULTS

Our samples for this study were collected from the granite complex at Fangshan pluton, Zhoukoudian, Hebei Province, China (Fig. 1). The granite body outcrops as a circle on the plane, which is 7.5–9 km in diameter. It is of medium size and was formed through two periods of magmatic activity during the time interval of 100 to 140 Ma according to isotopic tracer analysis (Tan and Ye, 1987). It was fuzzily classified into marginal, transitional, and central phases in space (Zhao, 2003), and from each phase, samples were collected as shown in Fig. 1 denoted by 1, 2, and 3 on the map.

The samples were identified and thin sections were prepared at the State Key Laboratory of Geological Processes and Mineral Resources, China University of Geosciences. Images of feldspar and quartz crystals on the sections were prepared digitally under plain polarization microscope with 10× ocular lens and 10× object lens, as shown in (A)–(F) of Fig. 2. One view of a square area of size 200×200 in pixels from each of the micro-images was selected and a 200×200 matrix of gray-scale values ranging from 1 to 256 was obtained correspondingly. For example, in Fig. 2, (a)–(f) are taken and magnified from (A)–(F),