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Soft hyperstructure

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ABSTRACT

In this paper, we study soft hypergroupoids. Firstly, we introduce the notion of soft hypergroupoids, and some examples are given. Also, we show that soft hypergroupoids are closely related to *L*-subhypergroupoid. Secondly, using the notion of soft hypergroupoid, some new properties of soft hypergroupoids are obtained. Lastly, we investigate some properties of soft subhypergroupoids.

1. Introduction

Hyperstructure theory was introduced [1] by Marty (1934) at the eighth congress of Scandinavian Mathematicians. The theory has been applied to other subjects of the classical pure mathematics by many researchers. Recently, comprehensive research has been done on the notion of hypergroupoid, hypergroup, hyperring and hypermodule; see [2–5]. Also, a recent book [3] contains a wealth of applications on geometry, hypergraphs, binary relations, lattices, fuzzy sets and rough sets, automata, cryptography, combinatorics, codes, artificial intelligence, and probabilistic. Another book [4] is devoted especially to the study of hyperring theory. Several kinds of hyperrings are introduced and analyzed. The volume ends with an outline of applications in chemistry and physics, analyzing several special kinds of hyperstructures: *e*-hyperstructures and transposition hypergroups.

Many complicated problems in economics, engineering, environment, social science, medical science and many other fields involve uncertain data. These problems which come face to face with in life cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the notion of the exact solution of this model is determined. Because of that the mathematical model is too complex, the exact solution cannot be found. There are several well-known theories to describe uncertainty. For instance fuzzy sets theory [6], rough sets theory [7] and other mathematical tools. But all these theories have their inherited difficulties as pointed out by Molodtsov [8]. Molodtsov introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties that is free from the difficulties affecting existing methods.

Soft set theory has rich potential for applications in several directions, few of which had been demonstrated by Molodtsov in his pioneer work [8]. At present, works on the soft set theory are making progress rapidly. Maji et al. [9] described the application of soft set theory to a decision making problem and studied several operations on the theory of soft sets. Pei et al. [10] discussed the relationship between soft sets and information systems. Roy et al. [11] described the application of fuzzy soft set theory to a decision making problem. Maji et al. [12] studied several operations on the theory of soft sets. Ali et al. [13] also studied some new notions such as the restricted intersection, the restricted union, the restricted difference, and the extended intersection of two soft sets. The algebraic structure of soft sets has been studied by some authors. Aktaş

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and Çağman [14] introduced the basic concepts of soft set theory, and compared soft sets to the related concepts of fuzzy sets and rough sets. They also discussed the notion of soft groups and drove their basic properties using Molodtsov's definition of the soft sets. Furthermore, Jun [15] introduced and investigated the notion of soft BCK/BCI-algebras. Jun et al. [16] discussed the applications of soft sets in ideal theory of BCK/BCI-algebras. In [17], Feng et al. introduced the notions of soft ideals and idealistic soft semirings, and gave several illustrating examples. In [18], Çelik et al. defined some new binary relations such as the extended sum, the restricted sum, the extended product, and the restricted product of soft sets, and derived their basic properties. In [19], Yang gave the notations of fuzzy soft semigroups and fuzzy soft (left, right) ideal, and discussed the α-level set, union and intersection of them.

In this paper, we will introduce the notion of soft hypergroupoids, which extends the notion of the hypergroupoids to include the algebraic structures of soft sets. We will deal with some properties of union, intersection, $\vee$-union and $\wedge$-intersection of the family of soft hypergroupoids.

2. Basic definitions

In this section, we will give some known and useful definitions and notations for the sake of completeness.

Molodtsov [8] defined the notion of a soft set in the following way: Let U be an initial universe set and E be a set of parameters. The power set of U is denoted by $\mathcal{P}(U)$ and A is a subset of E.

Definition 2.1 (*[8]*)**.** A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.

In [17], for a soft set (F, A), the set $Supp(F, A) = \{x \in A \mid F(x) \neq \emptyset\}$ is called the support of the soft set (F, A). If $Supp(F, A) \neq \emptyset$, then a soft set (F, A) is called non-null.

Zadeh [6] introduced the notion of a fuzzy subset μ of a non-empty set X as a function from X to $[0, 1]$. Goguen [20], generalized the fuzzy sets of X, to L-fuzzy sets, a function from X to a complete lattice L. For $\alpha \in L$, the set $\{x \in X \mid \alpha \leq \mu(x)\}$ denoted by μ_α is called an *α-level set*.

There is a close relation between L-fuzzy sets and soft sets (i.e., L-fuzzy subsets may be considered as a special case of the soft sets) as seen in the following example.

Example 2.2. Let X be a set, L be a complete lattice and $\mu : X \to L$ be an L-fuzzy subset of X. We define the set-valued function $F_\mu : L \to \mathcal{P}(X)$ defined by $F_\mu(\alpha) = \mu_\alpha$. Then the pair (F_μ, L) is a soft set over X. On the other hand if (F, L) is a soft set over X, then we can find an L-fuzzy subset $\mu_F : X \to L$ by means of the following formula: $\mu_F(x) = \bigvee_{x \in F(\alpha)} \alpha$ for all $x \in X$.

Similarly, there is a close relation between generalized rough lower and upper approximations and soft sets as seen in the following example.

Example 2.3. Let U and W be two universes. Suppose that R is an arbitrary relation from U to W. It can be defined as a set-valued function $F : U \to \mathcal{P}(W)$ by:

$$F(x) = \{y \in W \mid (x, y) \in R\}, \quad x \in U.$$

Obviously, the pair (F, U) is a soft set over W. In [21], for any set $A \subseteq W$, a pair of lower and upper approximations $\underline{R}(A)$ and $\overline{R}(A)$, are defined by

$$\underline{R}(A) = \{x \in U \mid F(x) \subseteq A\} \quad \text{and} \quad \overline{R}(A) = \{x \in U \mid F(x) \cap A \neq \emptyset\}.$$

Then the pairs $(\underline{R}, \mathcal{P}(W))$ and $(\overline{R}, \mathcal{P}(W))$ are soft sets over U, which is derived from the relation R.

Definition 2.4 (*[8]*)**.** Let (F, A) and (G, B) be two soft sets over U. Then,

(1) (F, A) is said to be a soft subset of (G, B), denoted $(F, A) \subseteq (G, B)$, if $A \subseteq B$ and $F(a) \subseteq G(a)$ for all $a \in A$,
(2) (F, A) and (G, B) are said to be soft equal, denoted $(F, A) = (G, B)$, if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$.

For a family $\{(F_i, A_i) \mid i \in \Lambda\}$ of soft sets over U, we can give some definitions as follows [13,17,22,12]:

- The *extended intersection* of the family (F_i, A_i) is defined as the soft set

$$(F, A) = \left(\bigcap_{\varepsilon}\right)_{i \in \Lambda} (F_i, A_i)$$

where $A = \bigcup_{i \in \Lambda} A_i$, $F(a) = \bigcap_{i \in \Lambda(a)} F_i(a)$ and $\Lambda(a) = \{i \mid a \in A_i\}$ for all $a \in A$,
- The *restricted intersection* of the family (F_i, A_i) is defined as the soft set

$$(F, A) = \bigcap_{i \in \Lambda} (F_i, A_i)$$

where $A = \bigcap_{i \in \Lambda} A_i \neq \emptyset$ and $F(a) = \bigcap_{i \in \Lambda} F_i(a)$ for all $a \in A$,