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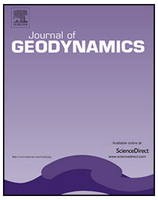

The relation between gravity rate of change and vertical displacement in previously glaciated areas

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ABSTRACT

The rate of change of surface gravity, $\dot{g}$, and vertical deformation rate of the solid surface, $\dot{u}$, are two observables of glacial isostatic adjustment (GIA). They contribute with different information on the same phenomenon. Their relation contains information of the underlying physics and a trustworthy relation allows to combine these observations to strengthen the overall observational accuracy of the phenomenon. In this paper we investigate the predicted relation between $\dot{g}$ and $\dot{u}$ in previously glaciated areas. We use the normal mode approach for one dimensional earth models and solutions of the sea level equation with time-dependent coastline geometry. Numerical predictions of $\dot{g}$ and $\dot{u}$ are computed for Laurentia, Fennoscandia and the British Isles respectively, using six different earth models. Within each region a linear trend is then fitted using the relation $\dot{g} = C\dot{u} + \dot{g}_0$. The estimated C and $\dot{g}_0$ differ more between the regions than between different earth models within each region. For Fennoscandia $C \approx -0.163\,\mu\text{Gal/mm}$ and for Laurentia $C \approx -0.152\,\mu\text{Gal/mm}$. Maximum residuals between the linear trend and spatially varying model predictions of $\dot{g}$ are 0.04 μGal/yr in Fennoscandia and 0.17 μGal/yr in Laurentia. For the British Isles the results are harder to interpret, mainly since this region is located on the zero uplift isoline of Fennoscandia. In addition, we show temporal variation of the relation since the last glacial maximum till present-day. The temporal and spatial variation of the relation between $\dot{g}$ and $\dot{u}$ can be explained by (i) the elastic respectively viscous proportion of the total signal and (ii) the spectral composition of the regional signal. Additional local effects, such as the Newtonian attraction and elastic deformation from local sea level changes, are examined in a case study for six stations in the Nordic absolute gravity network. The influence of these local effects on the relation between $\dot{g}$ and $\dot{u}$ is negligible except for extreme locations close to the sea.

1. Introduction

The ratio between the vertical displacement rate of the solid surface of the Earth, $\dot{u}$, and the rate of change of surface gravity, $\dot{g}$, has been shown to be useful when attempting to separate the present day ice mass (PDIM) change signal from the glacial isostatic adjustment (GIA) signal, the latter induced by historical ice mass variations, in regions like Greenland and Antarctica (Wahr et al., 1995; James and Ivins, 1998; Fang and Hager, 2001; Purcell et al., 2011; Memin et al., 2012). Given that the viscous part of the ratio as well as the elastic part of the ratio (including the direct attraction from surface mass variations) are known, simultaneous observations of $\dot{u}$ (e.g. GPS) and $\dot{g}$ (e.g. repeated absolute gravity observations) can be used to separate the delayed (viscous) signal from the instantaneous (elastic) signal (Memin et al., 2012). This proceeding is motivated by the fact that GIA models for Greenland and Antarctica contain uncertainties due to limited observations constraints (Purcell et al., 2011).

Mainly due to this purpose a number of investigations of the ratio between $\dot{u}$ and $\dot{g}$ have been published. Wahr et al. (1995) found that the viscous part of $\dot{g}$ is approximately proportional to the viscous part of $\dot{u}$ with the constant of proportionality $\sim\!-0.154\,\mu\text{Gal/mm}$ ($1\,\text{Gal} = 0.01\,\text{m/s}^2$). This approximation was based on empirical tests using a GIA model for Greenland and Antarctica, and was claimed to be insensitive to ice history and viscosity profiles in the mantle, which was later confirmed by Fang and Hager (2001). James and Ivins (1998) predicted $\dot{g}$ and $\dot{u}$ for Antarctica, using the ice model ICE-3G, and found their ratio to be

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Table 1
Published observations of $\dot{g}/\dot{u}$ in previously glaciated areas.

Area	$\dot{g}/\dot{u}$ [μGal/mm]	Note	Reference
Fenno.	-0.204 ± 0.058[a]	Relative gravity observations every 5th yr; time span ~27 yrs. $\dot{u}$ from mareographs and levelling.	Ekman and Mäkinen (1996)
Fenno.	-0.16 ± 0.05 to -0.18 ± 0.06[a]	Ekman and Mäkinen (1996) revisited, now with more observations. The different estimations of the ratio is related to different estimations of $\dot{u}$ (now including GPS).	Mäkinen et al. (2005)
Fenno.	-0.163 ± 0.02[b]	Four years of annual AG-observations on eight stations. $\dot{u}$ from GPS Lidberg et al. (2007). For the different stations the ratio varies between -0.114 ± 0.031 and -0.232 ± 0.059.	Gitlein (2009)
Fenno.	-0.17 to -0.22	13 stations with repeated AG observations compared to tide gauges data and GPS velocities	Pettersen (2011)
Laurentia	~ -0.154	Four stations of co-located GPS and AG. Total time span 6 yrs. Number of AG observations at the stations were 2, 2, 5, many. The ratio -0.154, from Wahr et al. (1995), is within the error bars of these observations.	Larson et al. (2000)
Laurentia	-0.18 ± 0.03[b]	Four stations of co-located GPS and AG. Three of the stations are the same as in Larson et al. (2000). Annual (at least) measurements in a time span of ~8 yrs.	Lambert et al. (2006)
Laurentia	-0.17 ± 0.01[b]	Eight AG stations whereof six are co-located with GPS, including the four stations in Lambert et al. (2006). Time spans 7–21 yrs	Mazzotti et al. (2011)

[a] 2σ (95% confidence interval).
[b] Type of accuracy not specified, probably 1σ.

~−0.16 μGal/mm. Purcell et al. (2011) studied the ratio between the viscoelastic load Love numbers h (describing the vertical displacement) and k (describing the gravitational potential change) in the spectral domain. This ratio depends on the harmonic degree and was here determined empirically from modelling.

In Laurentia in North America and Fennoscandia in northern Europe the situation is different. These regions were covered with ice during the Late Pleistocene but are long since ice free. Here the signal is a pure GIA signal (neglecting the small elastic response from sea level variations).

Fennoscandia has a long history of GIA observations in terms of e.g. sea level observations and levelling campaigns (Ekman, 1996), and during the last decades a lot of effort has been put in establishing a dense network of permanent GNSS stations (Scherneck et al., 2002) and co-located absolute gravity (AG) stations (Gitlein, 2009) in this region. Also in Laurentia a number of co-located GNSS and AG stations have been established (Mazzotti et al., 2011). One of the main long time goals of these efforts is to perform accurate observations of $\dot{u}$ and $\dot{g}$. Table 1 summarizes some published studies of the observed ratio $\dot{g}/\dot{u}$ in these regions. As the time series of continuous GNSS observations of $\dot{u}$ and repeated AG observations of $\dot{g}$ get longer and the observational accuracy increases, the question of their relation becomes prominent. Is a simple ratio accurate for relating geodetic observations of $\dot{g}$ and $\dot{u}$ in previously glaciated areas? The purpose of this paper is to investigate, via a modelling analysis, how robust a single relation between $\dot{u}$ and $\dot{g}$ is in previously glaciated areas, like Laurentia and Fennoscandia. Given a certain GIA model (described in Section 2) we predict $\dot{u}$ and $\dot{g}$ and show how their relation varies within each region and between the regions (Section 3). We also show, numerically, how it varies for different viscosity profiles in the earth model and how it varies in time since last glacial maximum (LGM) till present-day. Furthermore we investigate if additional effects from present-day sea level variations, like elastic deformation and direct attraction from the water masses, can be expected to affect the relation significantly (Section 4). Finally, we summarize the main findings in Section 5.

2. GIA-model

In Sections 3 and 4 a GIA-model is used to make predictions of $\dot{g}$ and $\dot{u}$. In this section the modelling method is indicated with references to more detailed descriptions, and relevant modelling parameters are presented. We also show some characteristics of the model since these will show important for the interpretation of the results in Sections 3 and 4.

The method used in the GIA-modelling is the normal mode approach for a one dimensional, laterally homogenous, spherical Maxwell Earth (Peltier, 1974, 1976, 1985; Cathles, 1975; Peltier and Andrews, 1976; Wu, 1978; Wu and Peltier, 1982, 1983). Specifically, our solution to the impulse response of a viscoelastic earth is expanded with the so-called collocation method, an approximation to the normal mode method proper. A critical evaluation of the two methods is found in Mitrovica and Milne (1992).

Based on the Preliminary Reference Earth Model (PREM) (Dziewonski and Anderson, 1981) viscoelastic load Love numbers (degree 1–180) have been computed using six different sets of earth model parameters (see Table 2). The digits in the model names represent lithospheric thickness [km], upper mantle viscosity [10^{21} Pa s] and lower mantle viscosity [10^{21} Pa s], respectively. The 96_0.5_10 compressible model is assumed to represent a realistic global average. The other models have been chosen so that the values for upper mantle viscosity and lithospheric thickness span a relatively broad range of values. We also considered a model that is identical to our reference model (96_0.5_10) except for that the elastic Lamé parameter was set very high to mimic the incompressible case.

The ice load history is defined by the ICE-5G model (Peltier, 2004) as included in the software SELEN 2.7 (Spada, 2003), i.e. 1 kyr time steps starting at the last glacial maximum (LGM) 21 kyr before present.

The response of the sea to the ice load changes has been computed by solving the sea level equation (SLE) (Farrell and Clark, 1976) with time-dependent coastline geometry following Mitrovica and Milne (2003) and Kendall et al. (2005). A more thorough description of our SLE solution can be found in Olsson et al. (2012).

With this definition of the GIA-model, the Earth's response to surface load variations is given. In order to understand the relation between $\dot{g}$ and $\dot{u}$, and how it varies in time and space, we will now examine some of the characteristics of the model.

Fig. 1 illustrates how the load Love numbers for earth model 96_0.5_10 depend on the spherical harmonic degree and time. In