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A new filter for the Mean Dynamic Topography of the ocean derived directly from satellite observations

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ABSTRACT

The Mean Dynamic Topography (MDT) of the ocean provides valuable information about the ocean's surface currents. Therefore the MDT is computed from satellite observations and then assimilated into ocean models in order to improve the ocean circulation estimates. However, the computation of the MDT from satellite observations of sea surface height and the Earth's gravity field is not straightforward and requires additional filtering of the data combination. The choice of the filter is crucial as it determines the amount of small-scale noise in the data and the resolution of the final MDT. There exist various approaches for the determination of an "optimal" filter. However, they all have in common the more or less subjective choice of the filter type and filter width. Here, a new filter is presented that is determined directly from the geodetic normal equations. By its construction, this filter accurately accounts for the correlations within the MDT data and requires no subjective choice about the filter radius. The new filtered MDT is assimilated into an inverse ocean model. Modifications in the meridional overturning circulation and in the poleward heat transports can be observed, compared to the result of the assimilation using the unfiltered MDT.

1. Introduction

The Mean Dynamic Topography (MDT) of the ocean is the difference between the Mean Sea Surface height and the geoid height, the geoid being an equipotential surface of the Earth's gravity field. The computation of the MDT is not straightforward because the different observational data sets have different representations and different resolution (Becker et al., 2012; Losch et al., 2002). Therefore, filtering becomes necessary in the MDT computation to remove small-scale noise.

Different approaches exist for the choice of the required filter[1] (Jekeli, 1981; Bingham et al., 2008; Jayne, 2006). A common choice is a Gaussian filter with an appropriate half-width radius. In Knudsen et al. (2011), a method is described for the determination of an "ideal" Gaussian filter width. Bosch and Savcenko (2009) promote an along-track filtering approach for the altimetric data and tolerate filter errors that arise from this one-dimensional filtering. An anisotropic filter is also used in Bingham et al. (2011) to filter the MDT. Filters that account for the error correlations of gravity field data are constructed e.g. in Swenson and Wahr (2006) and Kusche (2007). However, at the current stage, it is not clear which filtering is the most appropriate for the MDT.

In this study, we use the MDT error covariance matrix for the construction of a filter for the MDT data. The development of an MDT filter based on error covariances was already suggested in Bingham et al. (2008), however, its implementation depends on the availability of such an error covariance matrix. Here, the MDT estimate and its corresponding dense error covariance matrix described in Becker et al. (2012) are used.

The paper is organized as follows. An introduction to the MDT estimate and an introduction to the ocean model IFEOM are given in Sections 2.1 and 2.2, respectively. The derivation of the new filter is illustrated in Section 3.1. The filtered MDT and the filter residuals are compared to the results obtained by another filtering type in Section 3.2. The assimilation of the new filtered MDT into the ocean model IFEOM and a comparison of the results to those of the assimilation of the unfiltered MDT are presented in Section 4. A concluding discussion is provided in Section 5.

2. Background

2.1. Mean Dynamic Topography

The Mean Dynamic Topography (MDT) can be used to estimate ocean surface currents via the principle of geostrophy. Hence the combination of satellite observations of the sea surface height and

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[1] 'Filter' is used here in terms of mapping an input signal onto an output signal. It is not used in terms of LTI systems.

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the gravity field can reveal valuable information about the ocean's circulation (Wunsch and Stammer, 1998). However, satellite data of the MDT can only provide an incomplete picture of the ocean's state due to its two-dimensionality. Therefore in this study an MDT estimate is combined with an inverse ocean model in order to improve the understanding of the ocean's three-dimensional mean circulation.

For this purpose, a MDT was estimated from satellite observations by Becker et al. (2012). This MDT is designed exclusively for inverse ocean model assimilation. The MDT data $\boldsymbol{\eta}_\mathrm{d}$ and its corresponding inverse error covariance matrix $\boldsymbol{C}^{-1}$ are computed directly on an ocean model grid. The inverse error covariance is estimated from a least squares adjustment (geodetic normal equations) as described in Becker et al. (2012). This dense inverse MDT error covariance matrix is used as weighting matrix for the MDT model–data misfit in the ocean model optimization.

2.2. Inverse Finite Element Ocean Model (IFEOM)

The Inverse Finite Element Ocean Model (IFEOM) is a stationary model for the North Atlantic ocean (Sidorenko et al., 2006). It combines physical principles with observational data such as in situ temperature and salinity measurements and satellite data. This is accomplished by minimizing the cost function

$$J = \frac{1}{2}\sum_i J_i \overset{!}{=} \min, \quad \text{where } i = \text{MDT, temperature, salinity, etc.} \quad (1)$$

The different terms J_i contain quadratic model–data differences weighted by the inverses of their respective error covariances. Contributions from the residuals of the advection–diffusion equations for temperature and salinity are also contained in the cost function, so that the residuals are small. In this study, temperature and salinity data from a hydrographic atlas (Gouretski and Koltermann, 2004) are used for all IFEOM model runs. The MDT and its inverse error covariance matrix (Section 2.1) are assimilated in an unfiltered and in a filtered version.

In general, error correlations are unknown and diagonal inverse "covariance" matrices are used for weighting the different cost function terms. In our case, the full dense inverse error covariance matrix $\boldsymbol{C}^{-1}$ for the MDT data $\boldsymbol{\eta}_\mathrm{d}$ is provided by the approach described in Becker et al. (2012). Therefore the MDT term in the cost function (1) reads

$$J_\mathrm{MDT} = (\boldsymbol{\eta}_\mathrm{d} - \boldsymbol{\eta}_\mathrm{m})^T \alpha^{-1} \boldsymbol{C}^{-1} (\boldsymbol{\eta}_\mathrm{d} - \boldsymbol{\eta}_\mathrm{m}). \quad (2)$$

with the "observed" MDT $\boldsymbol{\eta}_\mathrm{d}$ from satellite data and their modeled counterparts $\boldsymbol{\eta}_\mathrm{m}$. The scalar factor α is derived from the Minimum Penalty Variance (MPV) approach (Freiwald, 2012) and is required for additional scaling.

The cost function (1) is minimized iteratively, starting from a first guess which is an earlier IFEOM solution described in Richter (2010). This first guess was computed using only the hydrographic data (temperature and salinity as described above), and therefore it is used here for a comparison with the model runs which assimilate MDT information. Details of IFEOM can be found in Sidorenko (2004) and Freiwald (2012).

3. A new filter based on the inverse error covariance

3.1. Construction

The inverse MDT error covariance matrix $\boldsymbol{C}^{-1}$ (Section 2.1) is used to construct the filter in order to account for the correlations in the MDT data.

In a first step, the matrix square root of $\boldsymbol{C}^{-1}$ is computed. This is possible and unambiguous because the inverse error covariance matrix is positive definite and symmetric by definition. In a second step, each row i of the resulting matrix $\boldsymbol{C}^{-1/2}$ is normalized. The corresponding normalization factors (not eigenvalues!) d_i are used to build the diagonal matrix $\boldsymbol{D}$:

$$\boldsymbol{C}^{-1/2} = \boldsymbol{D} \cdot \boldsymbol{S}. \quad (3)$$

For the computation of this decomposition, it has to be guaranteed that the diagonal entries d_i do not vanish. Due to the structure of the commonly used covariance matrices, this generally applies in applications: The covariance matrices have very large diagonals exceeding the off-diagonals by magnitudes, and therefore also the inverse and the inverse square root of a typical covariance matrix meet the condition.

The resulting matrix $\boldsymbol{S}$ from Eq. (3) has rows normalized to give a sum of one. This is necessary because the matrix $\boldsymbol{S}$ will be used to filter the MDT data $\boldsymbol{\eta}_\mathrm{d}$. The normalization ensures that the MDT is not reinforced or attenuated by the filtering process. This is equivalent to a weighted moving average filter with the weights given by the rows of $\boldsymbol{S}$, thus derived from the error covariances.

The unfiltered MDT $\boldsymbol{\eta}_\mathrm{d}$ and the filtered MDT $\boldsymbol{S\eta}_\mathrm{d}$ are shown in Fig. 1. Small-scale noise ("stripes") is largely removed by the filter $\boldsymbol{S}$ while oceanographic structures associated with strong currents, e.g. the Gulf Stream, are not considerably attenuated.

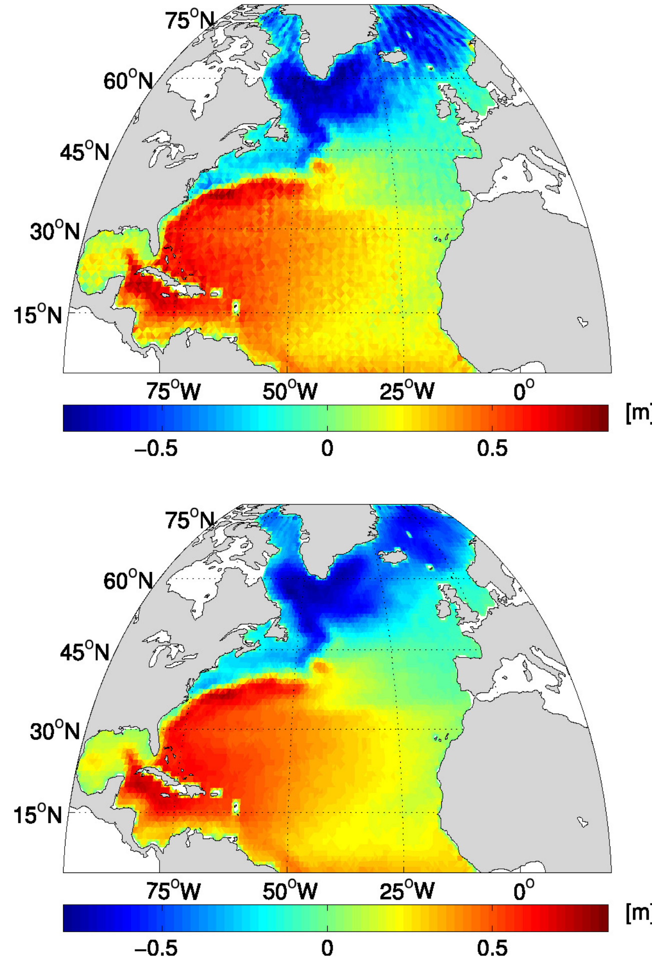

Fig. 1. Unfiltered satellite MDT estimate $\boldsymbol{\eta}_\mathrm{d}$ (top) and filtered MDT estimate $\boldsymbol{S\eta}_\mathrm{d}$ (bottom).

3.2. Comparison to simple moving average filters

In order to illustrate the advantage of this covariance-dependent filtering method, a comparison to the results computed with a