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Effect of different implementations of the same ice history in GIA modeling

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ABSTRACT

This study shows the effect of changing the way ice histories are implemented in Glacial Isostatic Adjustment (GIA) codes to solve the sea level equation. The ice history models are being constantly improved and are provided in different formats. The overall algorithmic design of the sea-level equation solver often forces to implement the ice model in a representation that differs from the one originally provided. We show that using different representations of the same ice model gives important differences and artificial contributions to the sea level estimates, both at global and at regional scale. This study is not a speculative exercise. The ICE-5G model adopted in this work is widely used in present day sea-level analysis, but discrepancies between the results obtained by different groups for the same ice models still exist, and it was the effort to set a common reference for the sea-level community that inspired this work. Understanding this issue is important to be able to reduce the artefacts introduced by a non-suitable ice model representation. This is especially important when developing new GIA models, since neglecting this problem can easily lead to wrong alignment of the ice and sea-level histories, particularly close to the deglaciation areas, like Antarctica.

1. Introduction

The Glacial Isostatic Adjustment (GIA), that is the delayed response of our planet to the loading history of the glaciation and deglaciation phases occurred in the past, is a phenomenon whose importance has been widely recognized. Reconciling the available data related to the palaeo-shorelines in the near and far field with respect to the glaciation centres, for example, requires a consistent modelling of the Earth response. On the other hand, direct information on the extent and amount of ice masses, and on the timing of the shorelines, allows us to better constrain the parameters characterizing the Earth interior (Barletta and Bordoni, 2009; Paulson et al., 2007; Wu and van der Wal, 2003; Mitrovica and Forte, 2004), that are mostly beyond any direct measurement. Such a knowledge of the Earth interior would be extremely important to make predictions about the near future. In fact, a renewed interest in GIA modelling, with special emphasis of the sea level variation related to GIA, comes just from the need for reliable predictions about the sea level variations due to the different *scenarii* of present day ice melting.

Since the fundamental work of Farrell and Clark (1976), where the equation describing the sea level variation due to GIA was cast, modified approaches have appeared in literature (Mitrovica and Peltier, 1991; Mitrovica and Milne, 2003; Spada and Stocchi, 2006, 2007) in order to make it suitable to describe more specific and special ice-water boundaries schemes (Johnston, 1993), or to include other solid Earth effects (Bills and James, 1996; Peltier, 1998; Wu, 2004) that were neglected in the original formulation. Nonetheless, the sea level equation still remains the same as defined in Farrell and Clark (1976).

This study focus on a specific issue in the solution of the sea level equation, related to the treatment of the ice loading history, that can have substantial effects on the predictions. The sea level equation is an integral equation which, in all cases of interests, can be solved numerically (and self-consistently). One fundamental ingredient in this equation is the ice loading history, which acts as an external forcing term. As it will be shown in the following, however, the solutions of the sea-level equation obtained by different implementations are equivalent if special care is taken of the way the load history is dealt with. All the most commonly used ice models for the last Pleistocene (glaciation and) deglaciation such as ICE-3G (Tushingham and Peltier, 1991), ICE-5G (Peltier, 2004), IJ05(Ivins and James, 2005), ANU (Lambeck et al., 2002), are provided as a set of snapshots of the ice mass distribution at given times. The specific representation of the ice mass distribution may

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differ, but for the purpose of the present work, the important feature is the time dependence of the ice load. All these models fall into two classes: the ones which assume a step-wise time evolution, that corresponds to depict the ice load history as a sequence of melting pulses at discrete times (ICE-XG), and the others that assume a linear piece-wise evolution between each time step (IJ05, ANU). The choice of the specific representation is up to the author of the ice model, and indeed reflects the approach used to derive the model. For what concerns the present discussion, these are simply two different ways of describing the main forcing term. The time representation of the load is an important part of the model specification. And the models are provided to be used in the exact form they are given. However, the details of the numerical solution algorithm require often the use of an "adapted" version of the ice model. For example, a code that cannot handle a linear ice history, needs the model to be mapped into a step-wise representation. In this case, the step-wise representation is an approximation of the original model, that is linear. But it is also possible that the algorithm requires piece-wise linear representation, and in that case a step-wise model has to be interpolated. In this case it is the linearized version that is an approximation to the original model. So neither the linear nor the step-wise version is in principle "better" than the other. The classification is in some sense not even especially meaningful, since the linear piece-wise representation can be considered the limiting case of a step-wise representation with infinitely small step size. And indeed, a more common situation is the one where the solution algorithm requires a specific form of step-wise representation (e.g. with regular time steps), and the models are given instead in a different step-wise form. This makes and a mapping into an internal representation necessary. Nonetheless, we keep in the discussion the distinction between linear and step-wise, since it helps in exemplifying the problem at hand.

In the following we focus on what happens when solving the sea-level equation using an ice model in a form that is not its original form. Even if the relation between the original and the adapted model reduces to a tuning of the step size, an important issue may occur when solving the sea level equation. This will be made clear through some simple examples, after a basic theoretical description.

2. Theory

A simplified picture can be useful to understand the GIA related sea level variation. A certain amount of ice mass melts from a formerly glaciated region, and the meltwater flows to the oceans. The mass distribution changes and the Earth responds to these variations both with an instantaneous (elastic) deformation, and with a delayed visco-elastic response. This is the reason for the present-day rebound measured for example in Hudson Bay and in Fennoscandia (Vermeersen and Schotman, 2009; Wu et al., 2010; Steffen et al., 2012), even if the last deglaciation ended there several thousand years ago. Notice that when the ice melts, the loading on the Earth changes in two ways: one is the change in ice mass, that is a change in mass localized on the glaciated land. The other, is the additional load on the oceans due to the meltwater redistribution.

As a consequence, the gravitational potential of the system "Earth + ice load + water load" changes, as well as the Earth surface topography. While the ice in this approximation is considered as independent loading force (and therefore indifferent to changes in potential and topography), water responds instantaneously to these variations, and therefore gives a "feedback", in the form of an additional mass variation, which in turn changes the Earth's gravity, until equilibrium is reached, and it enforces a delicate coupling between gravity and topography variations. The sea-level equation detailed in the following Section 2.1 describes just this phenomenon. This equation can be solved in different ways, e.g. the most commonly used method, the pseudo-spectral approach proposed by Mitrovica and Peltier (1991), the coupled Laplace-FE method (Wu and van der Wal, 2003; Wu, 2004) or the spectral-Finite element approach by Martinec (2000) and the consistent implementation of the sea-level equation into the last approach (Hagedoorn et al., 2006). Our method (and code) is based on the pseudo-spectral approach.

However, we want to stress that the equation depends crucially on two elements. The first is the Earth model, which determines how the Earth responds to a generic load. The second, is the Ice load history. These two elements are in principle completely unrelated, but unfortunately the data sets used to constrain the two models are not independent, and the ice load history itself is often at least partially constrained taking into account the sea level histories (Tushingham and Peltier, 1991; Peltier, 2004), through a suitable set of Earth parameters, making the problem a loop. Before going into further details, let us recall the basic theory of the sea-level equation, just to make clear the crucial role of the ice load term.

2.1. Sea level equation

By definition, the geoid is the equipotential gravitational energy surface at which the sea surface sits, at equilibrium. Of course, the sea level is determined also by the topography of the ocean basins, and in fact the sea level SL is defined as the difference between geoid height $r_{geoid}(\vartheta, \varphi)$ and topography $r_{topo}(\vartheta, \varphi)$:

$$SL(\vartheta, \varphi) = r_{geoid}(\vartheta, \varphi) - r_{topo}(\vartheta, \varphi). \tag{1}$$

During the glaciation and deglaciation phase, both Earth's gravitational potential and surface topography change. If N denotes the geoid variations and U the topography variations, the sea level variations become:

$$S(\vartheta, \varphi, t) = N(\vartheta, \varphi, t) - U(\vartheta, \varphi, t). \tag{2}$$

The sea level equation that provides the gravitationally self-consistent description of the GIA induced perturbation to the sea level can be written, after Farrell and Clark (1976):

$$S = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_O S - \frac{m_i}{\rho_w A_O} - \frac{\rho_i}{\gamma}\overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma}\overline{G_s \otimes_O S}. \tag{3}$$

Eq. (3) describes the space and time evolution of the sea level change $S(\vartheta, \varphi, t)$ in response to the ice load history $I(\vartheta, \varphi, t)$. ρ_i and ρ_w denote the ice and water density respectively, γ is the gravitational acceleration, G_s is the sea level Green function that represents the sea level change of the specific Earth model due to a generic unitary load and that, via spatial and temporal convolution $\otimes_i$, $\otimes_O$, provides the sea-level change for the specific load. m_i is the ice mass variation, A_O is the area of the oceans, and $\overline{(\ldots)}$ denotes spatial average over the ocean surface. For later reference, notice that the right hand side of Eq. (3) is made of two parts. The first represents the spatially varying component of the sea level variations $((\rho_i/\gamma)G_s\otimes_i I + (\rho_w/\gamma)G_s\otimes_O S)$. The last $(-(m_i/\rho_w A_O) - (\rho_i/\gamma)\overline{G_s\otimes_i I} - (\rho_w/\gamma)\overline{G_s\otimes_O S}$, sometimes denoted also as $C(t)$) instead represents a spatially uniform but time variable contribution, that guarantees mass conservation, and keeps track of the time variation of the reference geoid. The temporal derivative of this latter component, $\dot{C}(t)$, is the global uniform trend of sea level induced by GIA and it is used to correct altimetry data over the sea (Douglas and Peltier, 2002). As anticipated, S appears on both sides, due to the double role of the water. Notice in fact that the convolution is made both for the Ice term I and for S, though the subscripts i and O indicate that the spatial convolution is restricted to the geometry of the ice load and of the ocean basins respectively (Mitrovica and Milne, 2003). At a first approximation, the ocean area can be held fixed, i.e. the coastlines do not evolve