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Derivation of the radial gradient of the gravity based on non-full tensor satellite gravity gradients

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ABSTRACT

The formula for computing the radial gradient of the gravity is derived from the gravity gradients. Since GOCE satellite attitude has such property that one axis in the gradiometer almost has the same direction as the radial one of the earth, the formula can be used to compute the radial derivative of second order of the disturbing potential. This means that the boundary value problem (BVP) with the radial gradient of the gravity can be established on the satellite orbit. Hence, the recovery of the gravity field from GOCE data can be realized by solving the BVP. In order to examine the accuracy of the BVP, an arithmetic example is given, the computational results of which illustrate that the efficient degrees/orders of the gravity field model recovered by the BVP can satisfy the requirements of the satellite gravity gradients. In addition, a gravity field model is solved out from the BVP by using actual GOCE data.

1. Introduction

GOCE (Gravity field and steady-state Ocean Circulation Explorer) (Balmino et al., 1999; Albertella et al., 2002) is an explorer belonging to ESA (European Space Agency), and one of its main aims is to recover an accurate static gravity field model with high degrees and orders. GOCE has provided a huge amount of data about its position, attitude and the gravity gradients on the orbit since its launch on 17 March 2009. Obviously, it is widely concerned how to recover the gravity field models by using these data. The research focuses mainly on time-wise and space-wise approach.

Rummel et al. (1993) introduced the principles of the time-wise and space-wise approach. Keller (1997) investigated space-wise approach by constructing boundary value problems. Klees et al. (2000) proposed an efficient algorithm for recovering gravity field in time domain. Reguzzoni (2003) discussed the observation process from time domain to space domain. Albertella et al. (2004) analyzed the filtering techniques for gradients data processing in time and space domain. Milani et al. (2005) simulated the GOCE mission by time-wise approach whereas Migliaccio et al. (2007) conducted a simulation by space-wise approach. Reguzzoni et al. (2008) studied the application of step-wise solution in space domain. Pertusini et al. (2010) evaluated the structure of the covariance in space-wise solution. Pail et al. (2010) recovered the gravity field model GOCO01S using time-wise method by combining GOCE and GRACE data. The GOCE gravity field models from different methods were compared by Pail et al. (2011). This paper is aimed at doing some work in space domain, by constructing BVP.

Some notations are introduced here to make treatments clear. Let X_1, X_2 and X_3 be three axes of GRF (Gradometer Reference Frame), where the direction of X_3 is very close to radial one of the earth. Again letting v denote the gravitational potential of the earth, and $v_{ij} = \partial^2 v/\partial X_i \partial X_j$, v_{ij} $(i, j = 1, 2, 3)$ can be measured along the satellite orbit by the gradiometer and are called SGG (Satellite Gravity Gradients). Because of poor accuracies of v_{12} and v_{23}, only diagonal elements and v_{13} in SGG can be used to recover the gravitational field. Generally, recovery of the gravitational field from SGG can be attributed to solving the normal system of equations. The advantage of establishing the normal system of equations is that SST data (Satellite-to-Satellite Tracking) can be easily used to construct equations together with SGG data, but the issue is that the computation is very complicated since the unknowns exceed 40,000 if the degree of the model is higher than 200, although some researchers, such as Kusche (2002), Ditmar et al. (2003a), and Preimesberger and Pail (2003), investigated the methods of rapid resolution of GOCE gravity field models.

The approach by BVPs (Boundary Value Problems) with boundary condition of the radial gradients has its special advantage because of computing efficiently. In the early researches about GOCE, many researchers made deep discussions on BVPs. For example, Rummel and van Gelderen (1992) analyzed the spectral characteristics of $\{\Gamma_{zz}\}$, $\{\Gamma_{xz}, \Gamma_{yz}\}$, $\{\Gamma_{xx} - \Gamma_{yy}, 2\Gamma_{xy}\}$ and gave their two dimension Fourier expressions. van Gelderen and Rummel (2001) summarized the solution of the general geodetic BVPs by

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least-square methods and gave the solution under the boundary conditions of gravity gradients. Martinec (2003) discussed Green function to gradiometric BVP. However, SGG is not a full-tensor measurement because of the absence of v_{12} and v_{23}, thus it seemingly lacks some terms to establish BVP with the boundary conditions of the radial gradient. Recently, Yu et al. (2010) constructed the boundary conditions based on the invariants of the gravitational gradients, which can be represented as the radial component of the gravitational gradient tensor. This makes that BVPs can be used to deal with SGG data, which means that the model of the gravitational field can be recovered by the method of spherical harmonic analysis. The method of the invariants is not only simple, but affected with few attitude errors also. Consequently, Yu and Wan (2012) processed the actual SGG data and obtained a GOCE gravitational field model with higher than 200 degrees.

In fact, it is a key problem to derive the boundary conditions about the radial gradient in BVPs. Apart from the methods of the invariants (Sacerdote and Sansò,1989), the boundary condition of the radial gradient can also be derived from the coordinate transformation. However, in the actual GOCE data processing, the latter has not been paid enough attention. One reason may be that SST data can not be used as BVPs and a whole model of the gravitational model can not be recovered using only SGG data due to the limitations of the measure bandwidth. The other is that the radial gradient is different from v_{33} in the SGG so that it has to be composed of v_{ij} $(i, j = 1, 2, 3)$ while v_{12} and v_{23} can not be used due to their poor accuracies.

In the paper, it is discussed how to establish the boundary condition about the radial gradient from the coordinate transformation. Then the accuracy of the boundary condition is analyzed using the satellite attitude data. Finally, some arithmetic about recovery of the gravitational field from the boundary condition is given.

2. Establishing the boundary condition

2.1. Computation of the radial gradient

A reference gravitational potential V is chosen firstly, and then let $T = v - V$ be the disturbing potential. Generally, V should be a very good approximation to the actual gravitational potential v. For example, V can be chosen as EGM08 (Pavlis et al., 2012) or EIGEN_5C (Foerste et al., 2008) models. Letting α_1, α_2 and α_3 denote the angles between X_1, X_2 and X_3 in GRF and the radial direction of the earth, they can be computed from GOCE satellite attitude. By derivation, we have

$$\frac{\partial^2 T}{\partial r^2} = T_{33}\cos^2\alpha_3 + T_{22}\cos^2\alpha_2 + T_{11}\cos^2\alpha_1 + 2T_{12}\cos\alpha_1\cos\alpha_2 + 2T_{13}\cos\alpha_1\cos\alpha_3 + 2T_{23}\cos\alpha_2\cos\alpha_3 \quad (1)$$

where

$$T_{ij} = \frac{\partial^2 T}{\partial X_i \partial X_j} = v_{ij} - V_{ij}i, \quad (j = 1, 2, 3) \quad (2)$$

Since v_{ij} are measured from the gradiometer and V_{ij} can be computed from GOCE satellite's position and attitude, T_{11}, T_{22}, T_{33} and T_{13} are all known while T_{12} and T_{23} are unknown due to the poor accuracies of v_{12} and v_{23}. This means that $\partial^2 T/\partial r^2$ cannot be computed directly from Eq. (1).

On the other hand, although the actual X_3-axis is different with the radial direction of the earth, their difference is small. This means that $\alpha_1 \approx 90^0$, $\alpha_2 \approx 90^0$ and $\alpha_3 \approx 0^0$. Thus, it can be concluded that the term related to T_{33} is the main one in Eq. (1) and the terms related to T_{12} and T_{23} are small ones. To understand how small α_3 is, the attitude data of GOCE satellite during the period from 1 November to 10 November 2009 are chosen and the values of α_3 during the period are computed. Table 1 summarizes the statistics of α_3 and Fig. 1 gives its distribution during this period.

Table 1
Statistics of α_3 (unit: degree).

Number	Min	Max	Mean	Std
690,613	0.0008	1.3577	0.3859	0.2106

According to Fig. 1, angle α_3 is small and its maximum is less than 1.5^0. Hence, $\cos\alpha_3 > \cos 1.5^0 = 0.9997$, and $\cos\alpha_1, \cos\alpha_2 < 0.0262$ since $\cos^2\alpha_1 + \cos^2\alpha_2 + \cos^2\alpha_3 = 1$, that is, compared with $\cos\alpha_3$, $\cos\alpha_1$ and $\cos\alpha_2$ can be considered as small quantities. So, Eq. (1) can be written as

$$\frac{\partial^2 T}{\partial r^2} = T_{33}\cos^2\alpha_3 + T_{22}\cos^2\alpha_2 + T_{11}\cos^2\alpha_1 + 2T_{13}\cos\alpha_1\cos\alpha_3 \quad (3)$$

where the neglected magnitude is

$$\eta = 2T_{12}\cos\alpha_1\cos\alpha_2 + 2T_{23}\cos\alpha_2\cos\alpha_3 \quad (4)$$

In fact, η is the error of the boundary condition (3). Assuming that all the components in T_{ij} have the same magnitudes, the relative magnitude of the error η is approximately equal to

$$\delta = \frac{2\cos\alpha_1\cos\alpha_2 + 2\cos\alpha_2\cos\alpha_3}{\cos^2\alpha_3} \quad (5)$$

By computation, we know that $\delta < 5.5\%$. Also from Fig. 1, it can be seen that the average of the angle α_3 is less than 0.5^0. Hence, the relative error δ of the boundary condition (3) should be smaller. The value of δ given by report of ESA is 2% (Rummel and Gruber, 2012).

How to estimate the absolute error of η? This depends on the choice of the reference gravitational potential V. The more accurate the reference gravitational potential V is, the smaller the error η is. This is why EGM08 or ENGEN_5C are recommended as the reference gravitational potential. In addition, the iteration computations for T_{12} and T_{23} are also required, that is, T_{12} and T_{23} can be computed and then inserted into Eq. (1) after the disturbing potential T is solved firstly from the boundary condition (3). In fact, the effect for Eq. (1), caused by the absence of T_{12} and T_{23}, can be completely eliminated by such iteration process (Migliaccio et al., 2004a).

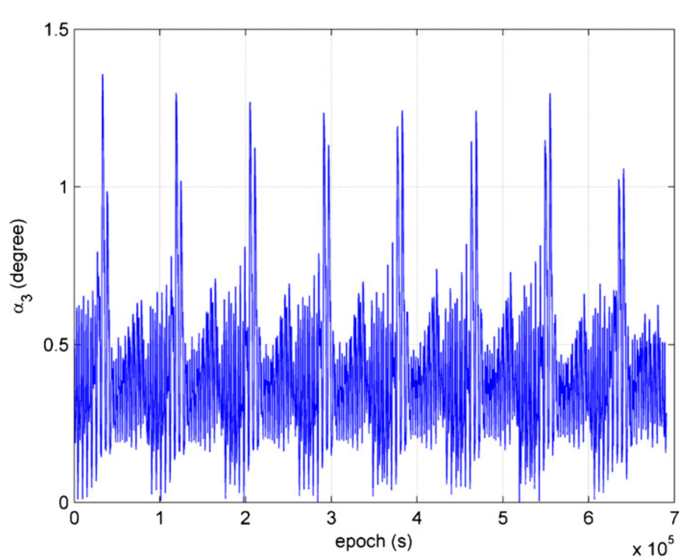

Fig. 1. Angle between X_3-axis of GRF and radial direction.