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Improved daily GRACE gravity field solutions using a Kalman smoother

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ABSTRACT

Different GRACE data analysis centers provide temporal variations of the Earth's gravity field as monthly, 10-daily or weekly solutions. These temporal mean fields cannot model the variations occurring during the respective time span. The aim of our approach is to extract as much temporal information as possible out of the given GRACE data. Therefore the temporal resolution shall be increased with the goal to derive daily snapshots. Yet, such an increase in temporal resolution is accompanied by a loss of redundancy and therefore in a reduced accuracy if the daily solutions are calculated individually. The approach presented here therefore introduces spatial and temporal correlations of the expected gravity field signal derived from geophysical models in addition to the daily observations, thus effectively constraining the spatial and temporal evolution of the GRACE solution. The GRACE data processing is then performed within the framework of a Kalman filter and smoother estimation procedure.

The approach is at first investigated in a closed-loop simulation scenario and then applied to the original GRACE observations (level-1B data) to calculate daily solutions as part of the gravity field model ITG-Grace2010. Finally, the daily models are compared to vertical GPS station displacements and ocean bottom pressure observations.

From these comparisons it can be concluded that particular in higher latitudes the daily solutions contain high-frequent temporal gravity field information and represent an improvement to existing geophysical models.

1. Introduction

Since March 2002 the twin satellite mission GRACE (Tapley et al., 2004) has measured the Earth's gravity field and its temporal variations with unprecedented accuracy. Various products and solutions have been presented in recent years. The standard procedure is the representation of the gravity field variations in terms of mean fields calculated over a certain time span resulting in monthly (Watkins and Yuan, 2007; Bettadpur, 2007; Flechtner et al., 2010), 10-day (Bruinsma et al., 2010) or weekly (Flechtner et al., 2010) solutions.

But there are various mass variation phenomena occurring on shorter time scales, for example the atmospheric variations or the barotropic motion of the ocean. In order to recover these fast gravity field variations as detailed as possible, it is reasonable to increase the temporal resolution with the goal of calculating daily GRACE solutions. This increase in temporal resolution, however, results in less observations per time span and therefore a reduced redundancy in the parameter estimation process. This leads to a decreasing accuracy of the estimated parameters with decreasing time span when the solutions are calculated individually. It can be assumed, however, that the gravity field does not change arbitrarily from one epoch to the next but that the information about the spatial and temporal correlation patterns can be approximated from geophysical models. Utilizing this knowledge, the temporal resolution can be enhanced without losing spatial information within the framework of a Kalman smoother estimation procedure. The Kalman smoother includes the stochastic prior information derived from geophysical models and the daily GRACE observations in a joint estimation process and delivers an updated state of the gravity field for each day. The stochastic information is introduced in terms of the process model which formulates a prediction of the current state resulting from the state of the previous time step. The process model is constructed from spatial and temporal covariance matrices derived from the output of the geophysical models.

In Kurtenbach et al. (2009) a first attempt was carried out to derive daily gravity field solutions from GRACE level-1B data. In this publication the assumption was made that the gravity field does not change at all from one day to the next. The uncertainty of this assumption was modeled in terms of a covariance matrix derived only from the hydrological model WGHM (Döll et al., 2003; Hunger and Döll, 2008). In this first approach a stationary, homogeneous, and isotropic stochastic process on the sphere was assumed. In the investigations presented here, an improved version will be introduced which takes into account the full correlation structure

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between two subsequent days, represented by the spatio-temporal covariance matrices. Besides the hydrological model, atmospheric and oceanic variations will be taken into account as well to derive a more realistic model of the process dynamic.

In Section 2 the Kalman smoother approach will be explained. In Section 3 a simulation scenario will be set up that uses geophysical models (different from those used to formulate the process model) to simulate daily GRACE observations for one year. This closed-loop scenario allows a variety of investigations to evaluate the potential of the method and to understand it in more detail. Especially the contribution of the GRACE observations to individual daily state updates will be investigated. In Section 4 the Kalman smoother approach will be applied to GRACE level-1B data resulting in daily GRACE solutions for the time span 2002–08 to 2009–08. These daily models are validated using individual data sets of ocean bottom pressure observations and vertical GPS station displacements.

2. The Kalman smoother approach

The temporal variations of the gravity field of the Earth can be approximated by a finite spherical harmonics expansion with a time dependant set of potential coefficients:

$$V(\lambda, \varphi, r, t) = \frac{GM}{R} \sum_{n=0}^{N} \left(\frac{R}{r}\right)^{n+1} \sum_{m=-n}^{n} a_{nm}(t)\, Y_{nm}(\lambda, \varphi), \tag{1}$$

with $V(\lambda, \varphi, r, t)$ being the gravitational potential at a given location (λ, φ, r) and point in time t, GM representing the Earth's gravitational constant, and R being the reference radius of the Earth. The Y_{nm} are the spherical harmonic functions and $a_{nm}(t)$ stand for the potential coefficients depending on the time t. In the following, these coefficients for each day will be estimated (Section 2.3) from a combination of daily GRACE observations (Section 2.1) and prior information about the temporal and spatial correlation patterns in terms of a process model which is derived from geophysical models (Section 2.2).

2.1. Observation model: GRACE level-1B data processing

The standard processing strategy used at the University of Bonn is based on the functional model described in, for example, Mayer-Gürr (2006) and Mayer-Gürr et al. (2007). It is based on a Fredholm integral equation of the first kind, which represents the solution of a boundary value problem to Newton's equation of motion formulated for short arcs of the satellite orbit.

The observation equations for the GRACE observations of one specific point in time t (i.e. a particular day) can be formulated as follows:

$$\mathbf{l}_t + \mathbf{v}_t = \mathbf{A}_t \mathbf{x}_t \quad \text{with} \quad \mathcal{C}\left\{\mathbf{l}_t, \mathbf{l}_t\right\} = \mathbf{R}_t, \tag{2}$$

with the GRACE observations $\mathbf{l}_t$, the residuals $\mathbf{v}_t$, the design matrix $\mathbf{A}_t$, the covariance matrix of the observations $\mathbf{R}_t$, and the unknown state vector $\mathbf{x}_t = (a_{nm})_t^T$, considered constant for one epoch (e.g. one day). From these assumptions it follows that the least-squares estimation procedure is solution of the normal equations:

$$\underbrace{\mathbf{A}_t^T \mathbf{R}_t^{-1} \mathbf{A}_t}_{\mathbf{N}_t} \mathbf{x}_t = \underbrace{\mathbf{A}_t^T \mathbf{R}_t^{-1} \mathbf{l}_t}_{\mathbf{n}_t}. \tag{3}$$

This reads explicitly

$$\hat{\mathbf{x}}_t = \mathbf{N}_t^{-1} \mathbf{n}_t. \tag{4}$$

However, the stand-alone daily solution of this ill-posed problem is very inaccurate due to a low number of observations and the dominant observation noise. Fig. 1 shows the gravity field solution for one arbitrary day using only one day of observations up to spherical harmonic degree $N = 40$ (top part). It becomes obvious that the solution bears little resemblance with the true gravity field features and that it is dominated by noise, which is confirmed by the very large standard deviation of the estimate shown in the bottom plot.

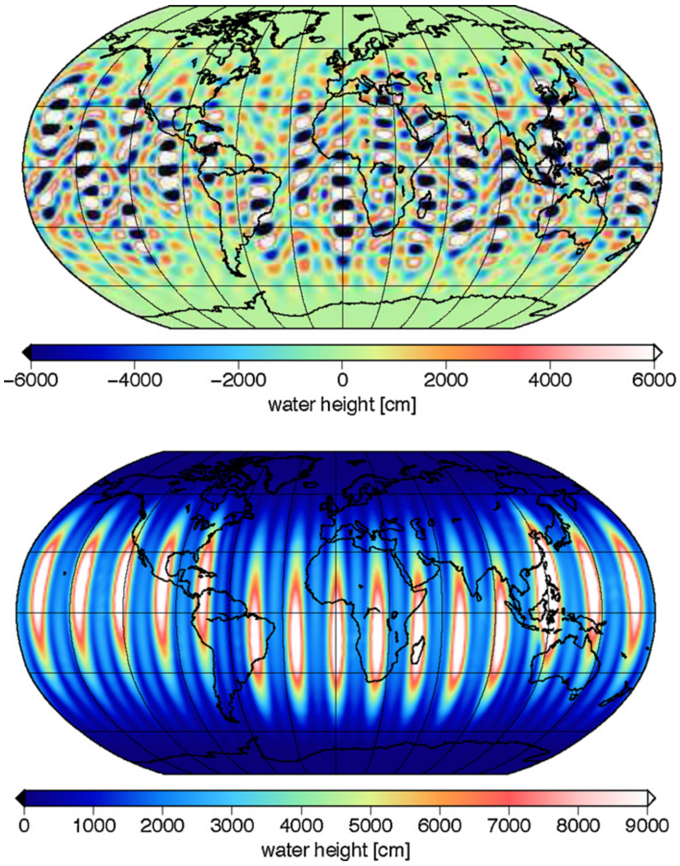

Fig. 1. One daily stand-alone GRACE solution (top) and its standard deviations in the spatial domain (bottom).

2.2. The process model

To obtain a reasonable gravity field model from one day of GRACE data, additional information is required. Up to now, no temporal correlations have been introduced into the processing strategy. But it can be safely predicted that the estimate of the gravity field will not change arbitrarily from one day to the next because temporal gravity field variations are driven by geophysical processes which are not random. Therefore one can assume the simplified case that the time evolution of the gravity field can be written as the following first order Markov process:

$$\mathbf{x}_t = \mathbf{B}\mathbf{x}_{t-1} + \mathbf{w}. \tag{5}$$

This represents a prediction of the gravity field coefficients from day $t-1$ to the current day t. The prediction is characterized by the matrix of the process dynamic $\mathbf{B}$. The normally distributed noise vector $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ with the covariance matrix $\mathbf{Q}$ represents the accuracy of the prediction.

In Kurtenbach et al. (2009) the simple assumption, that there are no changes between two states ($\mathbf{B} = \mathbf{I}$) with a covariance matrix $\mathbf{Q}$ derived from the hydrological model WGHM (Döll et al., 2003; Hunger and Döll, 2008), was introduced. Hereby a homogeneous and isotropic stochastic process on the sphere was assumed. In this section an improved version will be presented that takes into account the full correlation structure between two subsequent